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Hydraulic Modeling and Passivity-Based Control of Next Generation District Heating Networks

Bachelor's Thesis B433

Institut für Regelungs- und Steuerungssysteme
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A sustainable energy supply requires to rethink energy systems. In the course of a holistic energy transition, not only electrical power grids, but also gas and district heating networks have to be considered. Particularly so-called "4th or next generation heating networks" play an important role in fully decarbonizing densely populated areas. They are characterized by an increasing share of small, distributed heat generation units (DGUs) and pumps. However, the increased number of dynamically interacting subsystems (DGUs, pumps) is causing unclear hydraulic conditions (pressures, volume flows). Therefore, new control strategies and methods for pressure and volume flow control are necessary. Due to the large number of DGUs and their volatile character, the novel pressure and volume flow controllers should be designed in a decentralized manner.

At the IRS, first models and requirements for the decentralized control of hydraulics in next generation district heating networks have already been developed. A concrete controller design as well as a thorough stability analysis of the overall closed-loop system are however, as of now, missing.

Thus, within this thesis, decentralized controllers for the stabilization of pressures and volume flows in next generation heat networks are to be designed and a stability analysis of the overall hydraulic equilibrium is to be conducted. Starting point for the control design are generalized port-Hamiltonian system models of the hydraulic behavior of the relevant subsystems such as DGUs, end-users, and pipes. Based on these models, passivity-based controllers (e.g. using IDA-PBC or PI-PBC) for the actuators (pumps, valves) are to be designed. In the design process, the existing models are to be revised and extended if necessary (e.g. to include pressure holding units). During the thesis, the main focus can either be set on thorough theoretical results (model extensions, different controller designs, comprehensive stability analysis) or on more practical results featuring a validation in a small simulation environment (Simulink Simscape).

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I declare that I wrote my Bachelor's Thesis by myself and that I have followed the regulations relating to good scientific practice of the Karlsruhe Institute of Technology (KIT). I did not use any unacknowledged sources or means and I marked all references I used literally or by content.

Karlsruhe, 3rd May 2022

Raphael Stöckner

Abstract

In the 2015 Paris Climate Agreement, the goal was proclaimed to limit the global warming to 2 °C. This requires a drastic reduction of CO2 emissions in all sectors of energy production and consumption. Since half of the energy consumption in the European Union is used for heating or cooling and this sector thus plays an important role, the European Commission has created the Heat Map Europe. This large-scale study found out that the best way to reduce emissions in urban areas is to use district heating.

In district heating, water is pumped via a pipe system from heat generators to heat consumers. Whereas until a few years ago the producers were generally large power plants, renewable sources are now increasingly being used. In addition, different types of pipes are used, making it necessary to use pumps at the consumers as well.

These and other changes in the structure of heating networks are usually summarized under the terms "fourth and fifth generation district heating". While the networks can provide a renewable heat supply, some challenges arise with respect to their control. One of them appears on the hydraulic timescale, where the number of controlled actors increases drastically. With previous control methods, it is possible that the controllers affect each other negatively resulting in oscillations or even instabilities.

In this thesis, a solution for this problem is proposed. Existing models of heating networks are extended by a pressure control, a component that has been neglected in previous works and that can assure pressures in a safe operating range. Furthermore, controllers are developed that guarantee asymptotic stability of an equilibrium of the network in the presence of disturbances. In the manner of electric microgrids, controllers for a grid-feeding and a grid-forming mode are developed.

In a last step, the stability is analysed using passivity properties of the models and controllers. The analysis is carried out in a modular way that allows for different network structures and the plug in and plug out of both producers and consumers. The volatility of regenerative sources and the changing demand of the customers are therefore taken into account.

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Abbreviations and Symbols

Abbreviations

Abbreviation	Meaning
CHP	Combined heat and power plant
DGU	Distributed generation unit
DHN	District heating network
DHW	Domestic hot water
EEC	Electrical equivalent circuit
EU	European Union
EIP	Equilibibrium-independent passivity
HRD	Higher relative degree
IDA-PBC	Interconnection and damping assignment passivity-based control
ISO-PHS	Input-state-output port-Hamiltonian system
ME	Matching equation of IDA-PBC
MIMO	Multiple-input-multiple-output (system)
MPC	Model predictive control
PHS	Port-Hamiltonian system
PI-PBC	Proportional-integral passivity-based control
PnP	Plug-and-Play
RD1	Relative degree one
SISO	Single-input-single-output (system)
4GDH	Fourth generation district heating
5GDH	Fifth generation district heating

Latin letters

Symbol	Meaning
C	Capacity
$\boldsymbol{d} \in \mathcal{D} \subseteq \mathbb{R}^d$	Uncontrolled disturbance and interconnection input
$\boldsymbol{G}(\boldsymbol{x}) \in \mathbb{R}^{n \times m}$	Control input matrix
$H(\boldsymbol{x})$	Positive definite energy storage function or also *Hamiltonian* of the system, where $H : \mathcal{X} \rightarrow \mathbb{R}$
$\nabla H(\boldsymbol{x}) = \frac{\partial H}{\partial \boldsymbol{x}}(\boldsymbol{x}) \in \mathbb{R}^n$	Gradient of $H(\boldsymbol{x})$ w.r.t. $\boldsymbol{x}$
$\boldsymbol{J}(\boldsymbol{x}) \in \mathbb{R}^{n \times n}$	Interconnection matrix
$\boldsymbol{K}(\boldsymbol{x}) \in \mathbb{R}^{n \times d}$	Disturbance and interconnection matrix
L	Inductance
p	(Static) pressure

q	Volume flow rate
$\boldsymbol{R}(\boldsymbol{x}) \in \mathbb{R}^{n \times n}$	Dissipation matrix
$\boldsymbol{s}$	Transformed states for additional integral action
$\boldsymbol{u} \in \mathcal{U} \subseteq \mathbb{R}^{m}$	Control input
$\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^{n}$	State of the system
$\boldsymbol{y} \in \mathcal{Y} \subseteq \mathbb{R}^{m}$	Natural passive output
$\boldsymbol{z} \in \mathcal{Z} \subseteq \mathbb{R}^{d}$	Output conjugated to $\boldsymbol{d}$

Greek letters

Symbol	Meaning
$\boldsymbol{\beta}$	IDA-PBC control law
λ	Friction factor
$\boldsymbol{\phi}$	Parametrization of $\boldsymbol{R}(\boldsymbol{x})$ for parametrizing all passive outputs
$\boldsymbol{\omega}$	Function for parametrizing all passive outputs
ρ	Fluid density
Σ	Abbreviation for system

Calligraphic and other symbols

Symbol	Meaning
$\mathfrak{D}$	Set of DGUs in $\mathfrak{E}$
$\mathfrak{E}$	Set of edges in $\mathfrak{G}$
$\mathfrak{G}$	Dirgraph that represents a DHN
$\mathfrak{L}$	Set of consumers $\mathfrak{E}$
$\mathcal{L}$	The invariant set used for LaSalle's invariance principle
$\mathfrak{P}$	Set of pipes in $\mathfrak{E}$
$\mathfrak{V}$	Set of vertices in $\mathfrak{G}$

Indices, exponents and operator names

Symbol	Meaning
$\bar{(\cdot)}$	Unknown equilibrium
$(\cdot)^{*}$	Known equilibrium
$\tilde{(\cdot)}$	Derivation from an equilibrium
$(\cdot)_{\mathrm{c}}$	Customer substation
$(\cdot)_{\mathrm{form}}$	System 2 in grid-forming mode
$(\cdot)_{\mathrm{feed}}$	System 2 in grid-feeding mode
$(\cdot)_{\mathrm{l}}$	Pipes ("line")
$(\cdot)_{\mathrm{p}}$	System 1 ("pressure control")
$(\cdot)_{\mathrm{wD}}$	Parametrization of all passive outputs

Chapter 1

Introduction

1.1 Motivation and Background

In the past decades, the awareness about environmental problems with fossil-fueled energy supply has increased. Many new solutions have been proposed, either to replace environmentally unfriendly sources or to optimize existing alternatives. While topics like sustainable traffic or electrical power supply are excessively covered in science and media, other sectors of the energy market receive less attention but are important as well.

According to the European Union [Eur17] and as depicted in Figure 1.1, half of the energy consumed in the EU is used for heating or cooling purposes. While this consists in some parts to process heat in the industry, still more than 30% of the overall energy consumption is used either for space heating or for domestic hot water (DHW).

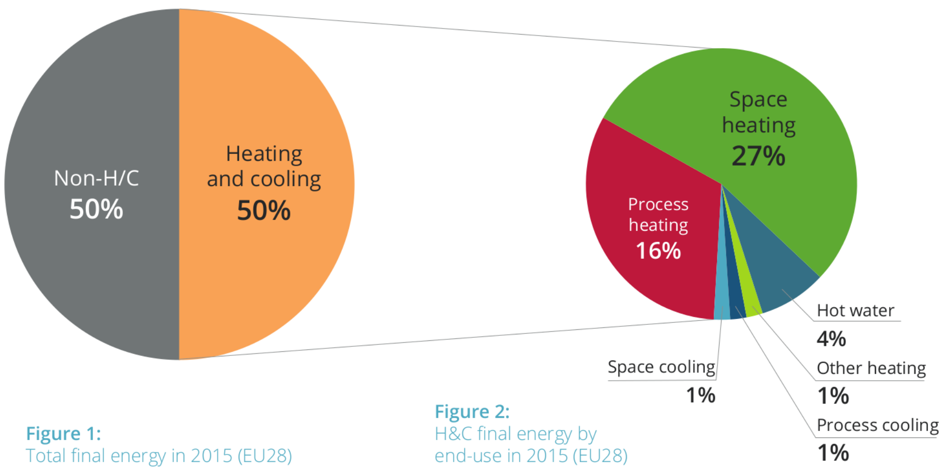


Figure 1.1: The rate of energy consumed in the EU for heating and cooling. Taken from [Eur17].

Due to this fact, the EU has created the Heat Roadmap Europe, a large project which aims at developing a sustainable heating infrastructure on the whole continent. Many different technologies have been analysed in the first step and one of the key components turned out to be district heating [Con+14; Paa+18].

District heating is a heating and cooling technology suitable for urban areas. The aim is to provide space heating and DHW to households in smaller areas at a local level. District heating networks (DHN) consist mainly of a pipe system which is fed with hot water by the producers. This hot water is pumped to the consumers who then can use the heat. Finally the cooled water flows back to producers where it is again heated. A schematic of a DHN can be seen in Figure 1.2.

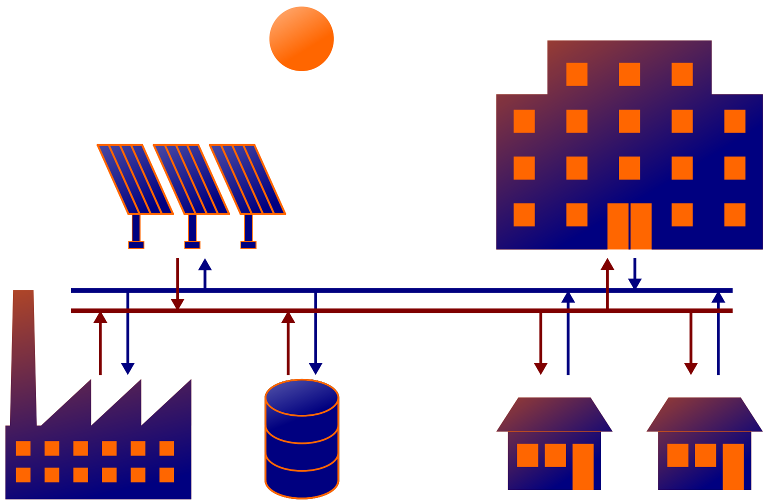


Figure 1.2: A schematic of a district heating network with different types of producers and consumers.

If the water is cooled down at the producer, a DHN can also provide efficient cooling. In the EU, this application scenario is not surprisingly mostly used in southern Europe [Eur17].

While the consumers have a rather homogeneous form - they are buildings like private households, public buildings or businesses - there are different forms of producers. The traditional way is to use gas powered plants [Lun+14], but many other systems such as combined heat and power (CHP) plants, usage of waste heat, solar-thermal heating plants or geothermal sources are also possible. Geothermal plants can further be used as storages. This concept was for example used in the Canadian city Calgary. In a flagship project, researchers could heat the city with, averaged over the year, 90% solar energy under use of storages that temporarily retained the energy [WM19].

The deployment of DHNs has several advantages. The results of [Paa+18, p. 12-27] show that with renewable energy sources and DHNs the decarbonization of the heating sector in urban areas is possible[1]. Additionally, they are very (physically) efficient and (financially) cost-effective systems. The researchers proclaim that the delivered energy can be reduced up to 25% by 2050 even under consideration of additional newly-built buildings.

Another advantage of DHNs is that from a geopolitical point of view, the usage of locally produced renewable energy allows independence from gas imports.

1.2 District Heating Networks of Fourth and Fifth Generation

The clear advantages of district heating have led to an increased research interest in the last decade. Although the technology is already mature and ready for deployment, a clear development towards more efficient networks is visible.

The evolution of DHNs can be categorized in five generations [Lun+14]. The first generation was already developed in the end of the 19th century and used steam as heat carrier. For the reason of large heat losses and severe accidents from steam explosions, the next generation refrained from steam. Instead, pressurized water was used albeit temperatures stayed above 100 °C. These systems were built until the 1970s and are rather rare today. Most nowadays operated systems belong to the

[1] In rural areas in contrast heat pumps are more efficient due to heat losses in the pipes of DHNs.

third generation. They are characterized by temperatures lower than 100 °C and mostly consist of prefabricated parts in order to minimize the effort at the construction site.

In the last decade, two new generations emerged which are called fourth and fifth generation district heating (4GDHN, 5GDHN) [Lun+14; Lun+21]. These are two different technologies which are developed in parallel and have several things in common.

Both generations use lower temperatures and therefore improve the efficiency by having lesser heat losses. The lower temperatures also enable the injection from low-temperature heat sources such as solar-thermal plants or waste heat. The newly made accessible sources further follow another aim: While earlier generations were powered mostly by fossil energy sources, current developments have the explicit goal of decarbonization.

These newly calibrated objectives lead to several changes in the network structure. The first three generations of DHNs relied on a fixed structure with one or few heat sources. In 4GDHN and 5GDHN in contrast, the energy generation becomes distributed and depends on much more sources, called distributed generation units (DGUs). This may be interpreted as that DHNs follow the evolution of other energy systems such as the electric grid towards a decentralized energy production.

The connection to other energy systems is likewise an important point as it does not just exist on a metaphorical level: There are huge efforts to make DHNs an integrated part of sustainable energy systems via combined heat an power (CHP) [VvH18; Her+18; Li+18; NH+19]. This technology tries to connect the electricity and the heating networks in a certain way that allows more flexibility. Depending on the current demand, the plant can produce heat for the DHN or electricity for the electric grid. If the input in the electric grid from other sources is high, maybe because regenerative sources have a temporal peak, the plant can also convert surplus energy into heat.

Nevertheless, there are some differences between 4GDHN and 5GDHN which shall be summarized briefly. 4GDHN hold the network temperature close to temperature demand at the customer. This leads to a better efficiency of the generation units and therefore low-temperature heat sources are able to deliver more heat at lower costs [MHH14; GZ13; AKY13]. The temperature is anyway high enough to meet the heat demands without additional heat pumps[2] or other systems. They can go as low as 45 °C without leading to a loss in comfort for the customers [TO18; Zü+18].

Another point that distinguishes 4GDHN from previous generations is the presence of hydraulic pumps at the consumer. This has been proposed in [Zin+08] and adopted in [Lun+14; DPK09; STD17; Str+21]. The reason for this is that during periods with low demand for DHW, little water flows through the pipes. The water therefore stays for a longer time in the pipe and cools down which causes the losses to become big. The solution is to split the pipe for hot water into two smaller pipes. One of them is the whole time in operation while the other one just carries water in times of high demand. The calculations show that this method can reduce the heat losses by up to 45% [Zin+08]. Unfortunately the results are a bit impaired by the higher pipe resistance due to the smaller diameters. The resistance causes the necessary pressures to rise which then demand for additional pumps.

An overview over the developments in district heating until the fourth generation is shown in Figure 1.3.

5GDHN, a term that was introduced in the FLEXYNET project [Fle], moves on to even lower temperatures. More precisely, the water temperature lies near to the ground temperature which requires heat pumps in every building. This may seem unintuitive, but also leads to efficient networks. Additionally, 5GDHN allow a bidirectional temperature flow between buildings and therefore a direct exchange of heat. This leads to a similar structure as in 4GDHN with additional pumps at the

[2] not to confuse with hydraulic pumps that are mentioned later on

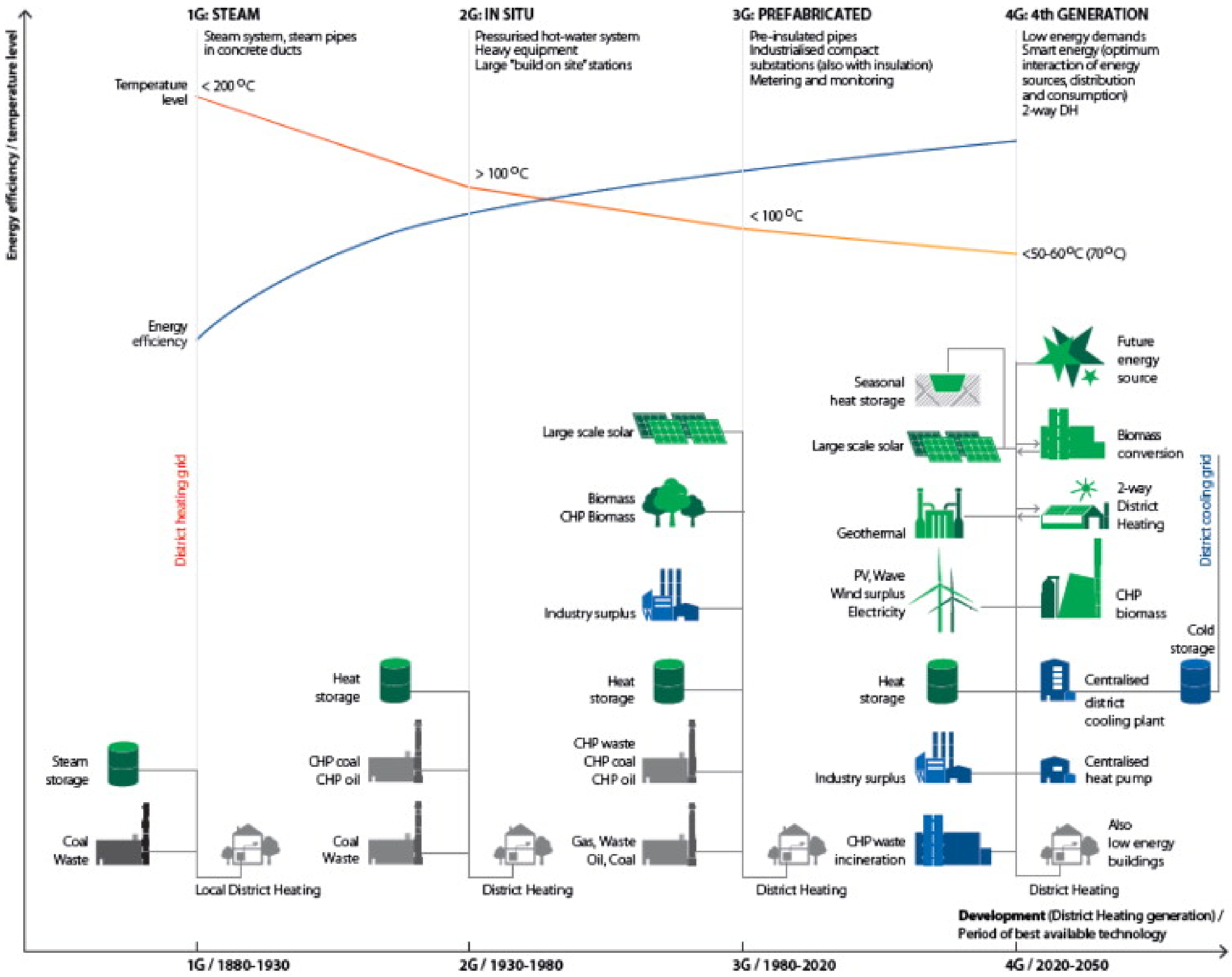


Figure 1.3: Comparison of the first four generations of district heating. Taken from [Lun+14].

consumer stations. Another modification compared to previous generations is the use of storages, e.g. borehole storages.

As a final remark, it should be noted that the term “generation”, although regularly used in literature, does not really fit: Due to the parallel development, 5GDHN are not a follow-up of 4GDHN. Instead, they pose two different, but similar ways for future development [Lun+21].

1.3 Control of DHNs

The developments of DHN poses new questions on how to control them. The huge number of independent pumps and controllers requires that explicit thought is given to the stability of the entire network. In the following, an insight on different aspects of the control of DHN is given with a special regard to hydraulic control. The section starts with regarding the state-of-the-art in Section 1.3.1. In Section 1.3.2, it continues both with the problems that exist today and those that will emerge in near future. Though few solutions for these problems exist in literature, the existing ones are presented in Section 1.3.3.

1.3.1 Current Control Concepts

Heating networks can be regarded in two timescales: The slow thermal timescale and the much faster hydraulic timescale. The reason is that the temperature travels mostly with the flow and is not transferred significantly between different fluid elements. The flow rate can be as low as 10 m/s and a changed supply temperature can need in the worst case hours until it is present in the farthermost parts of the network [CN19, p. 23]. The pressure in contrast is transferred between different fluid elements almost instantaneous. To be precise, it propagates with sonic speed [PGS16, p. 234].

Often the hydraulic timescale is consequently assumed as quasi-stationary and hydraulic and temperature are controlled separately. The advantage of this approach is of course its simplicity since it causes all the hydraulic dynamics to vanish when designing the thermal control law. Most control laws for DHNs use this approach, the ones that have been applied in practice for years as well as newer works in science. For reference, see e.g. [PGS16; QVZ21; vP20] and several of the controllers mentioned in [Buf+21]. At this point it may be noted that the thermal domain was and is the main research interest when it comes to new control methods for DHNs.

In practical applications, the separated control is realized by a static pressure regularization which controls the critical point with the lowest *differential pressure*. The differential pressure denotes the pressure between flow and return. It is necessary for letting the water flow and it has to be higher than the pressure losses at the substations of the consumers. The critical point in turn describes the point in the network where the differential pressure is minimal. This means that if the pressure is high enough at the critical point, it is high enough at every point in the network and the whole network has a state in which flows and pressure levels are within a desired range.

The critical differential pressure in conventional DHNs appears ideally at a well-known constant place. It is under this condition fairly easy to determine and to control [Lun+14]. The control takes place at the pump of the supplier and normally no advanced control methods are used. In [BAU19] for example, classic PID-controllers are used.

Once the pressure is set to a constant value, no other measurements are taken. As stated above, the hydraulic domain is assumed to be quasi-stationary, the pressure to be constant and as a direct follow-up the flow to be constant. The variations in heat demand are then handled by changing the amount of input heat at the producer. This is called the *central temperature control method* [Mad+94, p. 614] and is most commonly used in conventional district heating networks. As [VvH18] describes, the supply temperature can be determined dependent from the temperature outside.

1.3.2 Known Problems in Control and Future Challenges

Based on the assumption of quasi-stationarity of the hydraulic domain, many advanced temperature control systems have been proposed, e.g. [Her+18; Li+18; TO18; Zü+18]. They have different advantages compared to the central temperature control method such as that they can handle the temperature production more energy- or cost-effective. But all of them rely on the assumption that the hydraulic domain can be adjusted as needed without any problems.

It is a matter of fact that this approach has been successfully deployed in many different DHNs. But already today, some problems have been observed. First of all, the point of critical differential pressure can drift. This phenomenon can appear in conventional networks [Nus20, p. 56] and becomes worse in networks with multiple volatile producers, see [Bra+14, p.47] or [HE14; BAU19].

The simple reason for the drift can be illustrated by a small example: Suppose, that between a major producer like a CHP plant and the critical point is a small solar thermal installation. If the latter is close to the critical point, a small pump output at the solar thermal installation can lift the pressure *at* the critical point above the minimal pressure. Though, this small pressure boosting does not guarantee that the differential pressure at a point *behind* the original critical point is high enough. Therefore, the point of measurement is not the point of critical differential pressure any more and the critical point has shifted [Bra+14, p.47].

According to this logic, the situation becomes less overseeable with every additional volatile source. With an increasing number of sources, the pressure level has to be measured at many different points in the DHN. Following the thesis that current plans succeed and we will use more and more regenerative sources, this problem will eventually worsen.

Another problem in DHNs are oscillations which can have several negative effects. Problems are for example the generation of noise in the system, premature fatigue or even failure of components due to the violation of pressure limits. Oscillations can also lead to an instable temperature and pressure control [BT17].

One solution to address this problem is by using so-called differential pressure controllers. These are passive systems that are designed to keep the pressure over a control valve at the customers substations constant. Although widely used in practice, they can affect each other negatively [BT17]. Especially in areas with a high concentration of customers substations, there is an increasing risk of pressure oscillation, also called resonance oscillation. This effect appears when a differential pressure controller reacts to a pressure change from the network. Gradually, the surrounding pressure controllers react to this reaction until all of them act as one big instable differential pressure controller. This can happen even if most of them can individually be regarded as stable and balanced.

In Luleå in Sweden, another solution was found which mitigates pressure oscillations in DHNs [BAU19]. Plant engineers observed oscillations in temperature as well as in pressure. In this case, the underlying reason for the misbehaviour were not analysed due to the heavy experimental burden. Instead, a model of the whole DHN was created in Modelica using GIS databases and material constants. In a subsequent step, a model reduction was executed and a model predictive controller (MPC) was designed. A simulation in the realistic simulation framework OPTi-Sim [Opt] showed a decrease in temperature und pressure oscillations and an implementation in the real network in Luleå is planned.

This procedure via MPC is possible and gave good results. Yet, it also has several drawbacks: First of all, there is no strict proof of stability, even if it seems to work without problems in this case. Secondly, it also needs a permanent connection and data exchange between the different pumps in order to work. The pumps cannot work individually and the outage of a pump can lead to problems. And last but not least it has to be performed for every network individually and has to be repeated for example when the network is extended. As [BAU19] considers, sometimes dynamic models are also not available which makes the deployment of MPC approaches not just cumbersome but impossible. In this case, the oscillations have to be minimized in a different way.

Besides of the question of oscillations, another urgent point exists that has not been mentioned yet: Not only that a huge number of independent pumps and controllers can lead to oscillations, they even do not necessarily have to be stable. While an instable control loop is at least unlikely in a simple network with few pumps, this must not be the case for DHN of newer generations. One reason for this is that the network structure becomes *variable*: The pumps at the customer substations can plug in and plug out depending from the demand at the particular substation. The network structure is therefore continuously changing and the stability cannot be proven directly.

Several sources confirm that current control laws are not suitable for future 4GDHN and 5GDHN. Already in 2013, the AGFW observed in a report on trends in the heating sector [Paa+13, p. 266]:

> „Der Anschluss kleiner dezentraler Wärmeerzeugungsanlagen auf Basis erneuerbarer Energien erfordert in der Regel auch die Anpassung der Betriebsführung der zentralen

> Großanlagen, eine neue Regelstrategie des Wärmenetzes und ggf. auch die Netztransformation hin zu geringeren Netztemperaturen.“
>
> “The connection of small, decentralized heat generation units based on renewable energies usually requires an adjustment in the operational management of huge central plants, a new control strategy for heating networks and possibly the transformation of the network towards lower temperatures.”

Although some predictions as the trend towards lower temperature levels have already been taken up, few development of new controllers is visible for the hydraulic domain. Current control schemes are robust enough to guarantee the feed-in from a small number of sources, but they become inefficient with a rising number of producers [VvH18, p. 10].

1.3.3 Recently Proposed Control Schemes

As the last section shows, future DHN possess a number of problems that need to be solved. This reaches from hydraulic oscillations over the drift of critical point to the overall stability of the network and its operating point.

In this section, proposed solutions for these challenges shall be discussed and reviewed. As already mentioned, the field of hydraulic control of DHNs is poorly covered in research. Nevertheless, in recent years a small number control methodologies have been developed.

The results contain on the one hand fairly easy concepts as proportional controllers [DPK09] and PI-controllers [DP+14]. These two results use a *Plug-and-play* (PnP) approach. This means they model the subsystems of the DHN and find a control law that provides stability for the system independent from the interconnection structure of the subsystems. This allows that single subsystems can plug into and out of the network without affecting the stability, hence the name.

In these works, the system is modeled without any elasticities which already reduces the meaningfulness of the proposed solution. The results are further stymied by the fact that pumps are modeled as simple pressure sources without own dynamics. As the in DHNs commonly used [STD17] centrifugal pumps comprise a heavy pump wheel [Sig21, p. 32] with the corresponding moment of inertia, this modeling assumption neglects certain dynamics and abstracts the model strongly. Additionally, the concepts are designed for DHNs with only one producer and therefore are not applicable in modern heating systems.

[Jen12] delivers a series of papers dedicated to the control of hydraulic networks which were written in collaboration with de Persis, the author of the just referred papers. They follow a similar approach by using a PnP control scheme, similar models and a PI-control approach. Therefore they suffer from resembling problems.

A similar control law is derived in [STD17]. It is likewise a PI-resembling structure and follows the same modeling process. A speciality here is that an important property of centrifugal pumps is taken into account: They can create pressure and flow only in one direction. This is solved via a continuous, piecewise defined pump function and a strictly monotone Lyapunov function. A small disadvantage is that the control law is once again only suitable for DHNs with one heat source.

[Sch17b] derives controllers for different variations of DHN models. The first model contains storages with hot and cold water at every node in the network and will later in this work be called flow network (see Section 3.1.4). For this kind of model, controllers are developed for DHNs both with one producer as well as with multiple producers. Even so, the assumption of having a storage at every node is in practical terms rather unrealistic. Therefore another model is regarded which will later be denoted as hydraulic network and which does not have this disadvantage. For this kind of

DHN, however, only controllers for networks with one producer are presented [Sch17b, p.2, pp.5ff, pp.12ff.].

Multi-producer systems are considered in [TSD17]. Here, once again, flow networks are equipped with storages at the nodes. A mayor drawback, however, is that the pipe friction is neglected which has a considerable influence on the necessary pressure levels in the network.

Newer results published only this year come to the same conclusion as this work: A PI structure is a sensible controller structure as it is able to exploit the shifted passivity property of DHNs [Mac+22]. In consequence it will always render the controlled subsystem passive [Jay+07].

The control methods mentioned so far are all based on a modular PnP approach. This is advantageous for modern DHN since it already supports PnP by itself. Nevertheless completely different designs exist which shall be mentioned for the sake of completeness. As shortly mentioned in the previous section, [BAU19] proposes a MPC after a system identification and model reduction. [HKK03] proposes a nonlinear adaptive controller for mine ventilation systems, which have a very similar structure to DHNs. A problem that arises there is that all the controllers need information about every other controller and the system for information distribution can be complex and expensive. [KKSS06] proposes another nonlinear controller for the same purpose which bases on controlling the valves.

It strikes generally that none of the above-mentioned controllers for DHNs considers the pressure levels. All of them focus only on the flows in the hydraulic network. From a standpoint where the main goal of a DHN is to provide heat to the customers, this is understandable. As the heat travels mainly with the fluid elements and not between them, the adjustment of the correct flows is a central part in assuring a proper heat flow.

But the problem of the control of DHNs can also be seen from the point of view of an operator. In this case, the pressure levels are an equally important point for the reason that they guarantee a safe operation [Nus20, p. 54]. Too high pressure levels on the one hand can lead to leakage or even destruction of the pipes and hence have to be avoided. On the other hand, too low pressures can damage the pumps and drastically decrease their life span (ref. Section 3.1.6).

There are indeed actors in DHN that can directly manipulate the overall network pressure. They are known under the name *pressure control* and are widely used in practice. Additionally to keeping the pressure within admissible limits, they can compensate volume fluctuations of the fluid due to temperature fluctuations and water losses [Nus20, p. 54].

Considering the importance of the pressure control for the practice, it is surprising that no controllers for the pressure levels have been designed in research so far. This is a problem in the sense that the pressure control devices are additional active elements in the network. Therefore, they should contribute equally to the stability of the network as the flow pumps. It shall be mentioned at this place that the existing models of heat exchangers, pumps and pipes do not support the modeling of pressure control. Consequently, the existing models must first be put to the test before pressure control can be modeled.

This observation concludes this section. In the next one, these insights on the control of DHN are once again summarized. Subsequently, they are used to define the aim of this thesis and the requirements that have to be met.

1.4 Objective and Scope of the Thesis

The last section gave an overview over the various challenges that arise with the introduction of 4GDHN and 5GDHN. These appear both on the thermal and the hydraulic timescale and differ strongly from the problems that appeared up to now. However, on the thermal timescale, there is active research for new control methods and several viable solutions are already available. On the hydraulic timescale in contrary also problems exist, but they have not been solved satisfactorily yet.

The challenges for hydraulic control can be broken down to four main points:

C1 Oscillations in DHN are already present and will eventually worsen with the introduction of pumps at the customer substations.

C2 The current control objective of controlling the differential pressure becomes unfavorable as the point of critical differential pressure begins to drift. Again, this happens due to the introduction of pumps at the customer substations.

C3 The stability of the network has to be guaranteed under the consideration of many independent interconnected subsystems, in presence of plug-in and plug-out events and therefore under a changing network structure.

C4 The pressure levels have to be controlled everywhere in the network so that a secure operation is assured.

Of all the works, the results of [Sch17b] come closest to solving these problems. Even so, the network model with storages at the junctions may not meet a real DHN and no pressure control is possible. From this follows the motivation of this thesis and the overall objectives, which shall be refined and extended in the following.

Challenge C1 and C3 can be considered together as they both aim at stability and asymptotic stability, respectively. A control law that solves them both can take various forms. Advantageous in this context is a PnP approach as used in the majority of the literature, ref. Section 1.3.3. This allows the individual subsystems to be controlled independently of each other, and no consistent exchange of information is required between the various controllers [Las01]. To ensure stable operation nevertheless, a modular stability analysis must be conducted.

For the stability analysis, system characteristics of the individual subsystems are sought so that the stability of the interconnected system can be inferred from them. The characteristic that is particularly suitable in this case is passivity. It has already been successfully applied in a large number of electrical and power networks and ensures a scalable stability analysis. The details of the procedure and examples for the successful deployment are given in Section 2.3.2.

These considerations mean that two objectives can be deduced from the two challenges:

O1 The control of every subsystem is to be designed in a PnP fashion independently from other subsystems only with local parameters. To achieve this, it should use passivity features.

O2 The stability analysis shall be conducted in a modular way that ensures scalability and stability.

Furthermore C2 and C4 can be synthesized: They suggest that a control principle can be used that originates from electric microgrids.

Microgrids are a modern form of electrical networks that integrate renewable energy sources. Since the latter lack the inertia of big rotors in conventional power plants, they lack a mechanism that compensates for minor imbalances between supplied and consumed line on a very fast time scale. Therefore, a new control strategy had to be found and the principle of *grid-feeding* and *grid-forming* has been developed [Sch+16]. It operates the sources within the network in one of two operating modes: If the sources are operated in the grid-feeding mode, the power provided to the network is controlled. In grid-forming mode on the other hand, the stability of the network is secured by controlling the voltage in the microgrid and its frequency. Both modes operate usually only with local variables and the individual controllers represent freely composable modules.

	Electrical network	Heating network
grid-feeding	• Provide a desired power • Ensure the energy supply	• Set a volume flow • Ensure the heat supply
grid-forming	• Establish the power balance • Ensure stability	• Hold pressures within a certain range • Ensure secure operation

Table 1.1: Comparison of the control objectives of grid-forming and grid-feeding in electrical micro-grids and in DHN.

On an abstract level, this means that the grid-feeding mode ensures the functioning of the network and the grid-forming mode guarantees safe operation. This is the starting point according to which an equivalent controller for DHN can be designed:

The purpose of a DHN is to provide heat to the customers and the heat is transported by setting the volume flows. This means the functioning of a DHN is ensured by proper volume flows and a controller that fulfills this task can be considered a grid-feeding controller. On the other hand, a controller which keeps the pressure in the network within a desired range guarantees safe operation. Following the equivalence, it can be considered to be a grid-forming controller. If both types of controllers are present in a DHN, a secure and efficient operation is possible. The classification of the controllers and the correspondences between electrical and hydraulic domain is additionally shown in Table 1.1.

This concept has multiple advantages: First of all, it has already proven to be viable in electric networks. Moreover, it approaches the control problem in a modularized way as required in O1 and O2. And finally, there exist methods which can control the subsystems in a way that the modular stability analysis is possible and which only have to be adopted for DHN.

To the best of the authors knowledge, this is the first time that the pressure level in DHN is controlled. In addition, it is the first time that the classification of grid-forming and grid-feeding is transferred to the hydraulic domain.

The control structure adds two more point to the set of objectives:

O3 Ensure the functionality of the DHN by designing a grid-feeding controller which controls the volume flow at every substation.

O4 Ensure the safe operation of the DHN by keeping the pressures within a secure range using a grid-forming controller.

With these points, the objectives of this work are clearly defined. After a short overview of the notation used in this work, the rest of the thesis is structured as follows: In Chapter 2, the necessary foundations for the work are given and it is explained how a modular stability analysis can work. The models are given in the following Chapter 3. Additionally, they are discussed and important system properties will be emphasized. The latter ones pave the way towards Chapter 4 where the controllers are finally designed and the stability analysis is conducted. The work closes with an assessment of the control, a comparison with controllers of different domains and some final remarks.

1.5 Mathematical Notation

Throughout the thesis, the mathematical notation will be as follows: Vectors are considered to be column vectors and they are written in lower case and in bold, e.g. $\boldsymbol{x}$. Matrices are also written in bold letters, but in upper case as the matrix $\boldsymbol{A}$. The gradient of a scalar function w.r.t a vector $\boldsymbol{x}$ results in a column vector and can be written shorthand by

$$\nabla H(\boldsymbol{x}) = \frac{\partial H(\boldsymbol{x})}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial H(\boldsymbol{x})}{\partial x_1} \\ \frac{\partial H(\boldsymbol{x})}{\partial x_2} \\ \vdots \\ \frac{\partial H(\boldsymbol{x})}{\partial x_n} \end{pmatrix}.$$

The indices mean the following: $\boldsymbol{x}^*$ represents a known equilibrium and $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}^*$ is the derivation of the equilibrium, also called the incremental variable. An unknown equilibrium is denoted wit $\bar{\boldsymbol{x}}$ and a vector that consists of both known and unknown equilibria is denoted as $\bar{\boldsymbol{x}}^*$. The index d denotes a desired variable. In the course of this work, variables, matrices and functions like $\boldsymbol{x}$, $\boldsymbol{R}$ or $H(\boldsymbol{x})$ appear multiple times for different systems. The related system is not further marked since in every section only one system is treated. The attribution to the respective system is therefore clear.

Chapter 2

Fundamentals

After the statement of the problem and the purpose of this work, the necessary foundations are presented. The chapter starts with an introduction of Lyapunov stability theory in Section 2.2 followed by the thereon based passivity theory in Section 2.3. These facts are used to illustrate the special characteristics of Port-Hamiltonian system modeling in Section 2.4.3. The chapter closes with Section 2.5 in which an overview over nonlinear control is given with a special regard to passivity-based control.

2.1 Mathematical Fundamentals

In this section, some definitions and propositions are stated that are necessary for the following work. The section starts with the definition of positive semidefinite matrices and a method to ensure their existence in Section 2.1.2. The following Section 2.1.1 furthermore treats symmetry properties of matrices.

2.1.1 Symmetric and Skew-symmetric Matrices

The properties of a matrix mapping can be characterized in several ways. Fundamental properties are symmetry and skew-symmetry [Sho18, p. 110, 117], as man other characteristics can be inferred from it.

Definition 1 (Matrix symmetry): A matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is called *symmetric*, if $\boldsymbol{A} = \boldsymbol{A}^\top$. Furthermore it is called *skew-symmetric*, if $\boldsymbol{A} = -\boldsymbol{A}^\top$.

Skew-symmetry has a useful property when a matrix is multiplied from both sides:

Lemma Suppose a matrix $\boldsymbol{A} = (a_{ij}) \in \mathbb{R}^{n\times n}$ is skew-symmetric. With an arbitrary vector $\boldsymbol{x} = (x_i) \in \mathbb{R}^n$ follows

$$\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} = 0 \tag{2.1}$$

■

Proof The proof is straightforward:

$$\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} = \sum_{i=1}^{n} x_i \sum_{j=1}^{n} a_{ij}x_j = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_ix_j = 0$$

since $x_ix_j = x_jx_i$ and due to the definition $\boldsymbol{A} = -\boldsymbol{A}^\top$ the every element a_{ij} eliminates the element a_{ji}. ■

2.1.2 Positive and negative semidefinite matrices

Another important criterion to characterize matrices is the definiteness of a matrix [Bha07, Ch. 1.1].

Definition 2 (Semidefiniteness of Matrices): A matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is called *positive semidefinite*, if for every $\boldsymbol{x} \in \mathbb{R}^n$ the statement

$$\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{0} \tag{2.2}$$

holds. One writes $\boldsymbol{A} \succcurlyeq 0$. Equivalentely it is called *negative semidefinite*, if for every $\boldsymbol{x} \in \mathbb{R}^n$

$$\boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{0} \tag{2.3}$$

holds. In this case, one writes $\boldsymbol{A} \preccurlyeq 0$.

An obvious criterion on the semidefiniteness are the eigenvalues of the matrix, but in this work, a more feasible condition is used, called Sylvester's criterion. In order to formulate it, the principal minors of a matrix have to be defined [Min].

Definition 3 (Minors of a Matrix): The determinant of a submatrix $\boldsymbol{B} \in \mathbb{R}^{k\times k}$ of a matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}, n \geq k$ is called a *principal minor* of $\boldsymbol{A}$, if the following holds: $\boldsymbol{A}$ equals $\boldsymbol{B}$ if $n-k$ columns and rows of $\boldsymbol{A}$ are deleted. Additionally, the indices of the deleted columns and rows are equal.

With this definition, Sylvester's criterion follows as [Swa73]:

Theorem 1 (Sylvester's criterion): A real, symmetric matrix $\boldsymbol{A}$ is positive semidefinite, iff all of its principal minors are nonnegative. Analogously, it is called negative semidefinite, iff all of its principal minors are nonpositive.

2.2 Lyapunov Theory

Stability is arguably the most important property in control theory and different approaches and definitions exist. The classical control theory has been developed in the western hemisphere by Nyquist and Bode [SJK12, pp. 2ff.] around the time of the Second World War. It regards the frequency domain of a system and is applicable to linear, time-invariant systems. Besides, another branch of stability theory exists that has been founded by Poincaré and Lyapunov. This theory bases on considerations in the time domain and regards stability as the stability of a certain equilibrium.

In order to define the Lyapunov stability property, a nonlinear state system can be considered [Kha02, p. 202] with the form

$$\Sigma := \begin{cases} \dot{\boldsymbol{x}} &= \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y} &= \boldsymbol{g}(t, \boldsymbol{x}, \boldsymbol{u}) \end{cases} \tag{2.4}$$

with $\boldsymbol{x} \in \mathbb{R}^n = \mathcal{X}$, $\boldsymbol{u} \in \mathbb{R}^m = \mathcal{U}$, $\boldsymbol{y} \in \mathbb{R}^q = \mathcal{Y}$, $\boldsymbol{f} : [0, \infty) \times D \times D_u \to \mathcal{X}$ resp. $\boldsymbol{g} : [0, \infty) \times D \times D_u \to \mathcal{Y}$ piecewise continuous in t and locally Lipschitz continuous in $(\boldsymbol{x}, \boldsymbol{u})$, $D \subset \mathbb{R}^n$ contains $\boldsymbol{x} = \boldsymbol{0}$ and $D_u \subset \mathcal{U}$ contains $\boldsymbol{u} = \boldsymbol{0}$.

Stability can now be defined as follows [Sch17a, p. 44]:

Definition 4 (Lyapunov stability): Let $\boldsymbol{x}^*$ be an equilibrium of Eq. (2.4), which means

$$\boldsymbol{f}(t, \boldsymbol{x}^*, \mathbf{0}) = \mathbf{0} \text{ and } \boldsymbol{x}(t, \boldsymbol{x}^*) = \boldsymbol{x}^*$$

for all t. This equilibrium is called

- *stable*, if for each $\varepsilon > 0$ there exists $\delta(\varepsilon)$ such that

$$\|\boldsymbol{x}_0 - \boldsymbol{x}^*\| < \delta(\varepsilon) \Rightarrow \|\boldsymbol{x}(t, \boldsymbol{x}_0) - \boldsymbol{x}^*\| < \varepsilon, \qquad \forall t \geq 0 \tag{2.5}$$

- *asymptotically stable*, if it is stable and additionally a $\bar{\delta} > 0$ exists such that

$$\|\boldsymbol{x}_0 - \boldsymbol{x}^*\| < \bar{\delta} \Rightarrow \lim_{t \to \infty} \boldsymbol{x}(t, \boldsymbol{x}_0) = \boldsymbol{x}^* \tag{2.6}$$

- *globally (asymptotically) stable* if it is (asymptotically) stable for all $\boldsymbol{x}_0 \in \mathcal{X}$
- *unstable* if it is not stable.

Loosely speaking, stability is the property that the state $\boldsymbol{x}$ stays within a certain range for every $\boldsymbol{u}$ and t. Asymptotic stability is a bit stricter and means that $\boldsymbol{x}$ will eventually converge towards the equilibrium $\boldsymbol{x}^*$.

Lyapunov's Stability Theorem

In order to establish Lyapunov's stability theorem, two terms have to be established.

Definition 5 (Positive definite functions): A function $f : \mathbb{R}^n \to \mathbb{R}$, $n \in \mathbb{N}$ is called *positive definite* [How10, p. 6] if

$$f(\boldsymbol{x}^*) = 0 \qquad \text{and} \qquad f(\boldsymbol{x}) > 0, \quad \boldsymbol{x} \neq \boldsymbol{x}^* \tag{2.7}$$

with $\boldsymbol{x}^* \in \mathbb{R}^n$ being an arbitrary point. In analogy, it is called *negative definite* if

$$f(\boldsymbol{x}^*) = 0 \qquad \text{and} \qquad f(\boldsymbol{x}) < 0, \quad \boldsymbol{x} \neq \boldsymbol{x}^*. \tag{2.8}$$

Semidefiniteness is a relaxed property that is defined equally only with admitting equality in Equations (2.7) and (2.8), i.e. replacing $>$ and $<$ with $\geq$ and $\leq$.

Definition 6 (Lyapunov functions): A C^1 function $V : \mathcal{X} \to [0, \infty)$ serves as a *Lyapunov function* [Sch17a, p. 44] for an equilibrium $\boldsymbol{x}^*$ of Eq. (2.4) if

- it is a positive definite function with regard to $\boldsymbol{x}^*$
- as well as

$$\dot{V}(\boldsymbol{x}) = \nabla V^\top(\boldsymbol{x})\dot{\boldsymbol{x}} = \nabla V^\top(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) \leq 0, \qquad \boldsymbol{x} \in \mathcal{X}. \tag{2.9}$$

Theorem 2 (Lyapunov stability theorem): Let $\boldsymbol{x}^*$ be an equilibirum of Eq. (2.4). If a Lyapunov function exists for the equilibrium $\boldsymbol{x}^*$, then it is a stable equilibrium in the sense of

Definition 4. If additionally

$$\dot{V}(\boldsymbol{x}) < 0, \qquad \forall x \in \mathcal{X}, \qquad x \neq \boldsymbol{x}^*, \tag{2.10}$$

then $\boldsymbol{x}^*$ is an asymptotically stable equilibrium.

A Lyapunov stability analysis always works only in one direction: It can only prove stability but never prove instability. Whether an analysis is successful therefore depends from a proper selection of a Lyapunov function. This can be in general a quite exhaustive and complicated search. Fortunately, some strategies haven proven to be successful: Often the energy stored in the system is used as a Lyapunov function. Due to energy conservation, this mostly leads to a viable stability analysis in the case of systems without active elements. Usually, the functions are then called energy functions or storage functions and at a suitable place in the analysis they are used as Lyapunov functions.

Remark 1: A physical system always tries to reach the state with minimal energy. This justifies and reasons Theorem 2 and the approach over the system energy: In order to reach the minimum, the stored energy must decrease at every point besides the minimum. The energy cannot become negative and neither does the Lyapunov function.

LaSalle's Invariance Principle

The generality of Lyapunov's stability theorem to apply it on every possible system is a great advantage, but regrettably it does not always yield a full solution. In this case, another theorem has to be applied, the invariant set theorem of LaSalle [Sch17a, p. 45].

Definition 7 (Invariant sets): A set $\mathcal{N} \in \mathcal{X}$ is called *invariant* for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ if $\boldsymbol{x}(t, \boldsymbol{x}_0) \in \mathcal{N}$ for all $\boldsymbol{x}_0 \in \mathcal{N}$ and $t \in \mathbb{R}$. It is called *positively invariant*, if the statement holds only for all $t \geq 0$.

Theorem 3 (LaSalle's Invariance Principle): Let $V : \mathcal{X} \to \mathbb{R}$ be a C^1 function for which $\dot{V}(\boldsymbol{x}) = \nabla V^\top(\boldsymbol{x})\dot{\boldsymbol{x}} \leq 0, \forall x \in \mathcal{X}$. Additionally suppose that there exists a compact set $\mathcal{C}$ which is positively invariant for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$. Then for any $\boldsymbol{x}_0 \in \mathcal{C}$ the solution $\boldsymbol{x}(t, \boldsymbol{x}_0)$ converges for $t \to \infty$ to the largest subset of $\{x \in \mathcal{X} | \dot{V}(\boldsymbol{x}) = 0\} \cap \mathcal{C}$ that is invariant for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$.

Remark 2: The invariance principle can be seen the following way: It has already been proven that the system is "somewhat" stable as it has been shown that the state will always result in the set of states with (at least locally) minimal energy. For the states to be a possible equilibrium, another condition has to be met, namely that the state is constant over time. If this is not the case, the system may have a minimal stored energy, but every time it enters this state, it will immediately leave it. LaSalle's Theorem now tells that a state has to be the equilibrium of the system if it is the only state state that lies in the intersection of two sets: The set of states with minimal energy and the "set of states that stay within that set", hopefully constant states. If multiple states lie in the intersection, the theorem still holds but it does not finish the stability proof.

Zero-state Observability

The following definition regards the behaviour of states under input and output equal to zero [Kha02, Def. 6.5]:

Definition 8: A system of the form (2.11) is said to be zero-state observable, if no solution of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{0})$ can stay identically in $S = \{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{0}) = \boldsymbol{0}\}$, other than the trivial solution $\boldsymbol{x} = \boldsymbol{0}$.

Remark 3: LaSalle's invariance principle cannot be applied on systems that are not zero-state observable.

Remark 4: There are also other definitions of zero-state observability with slightly different implications, see e.g. [SFM19, p. 17f.]. In this work, however, the definition from the standard reference [Kha02] is used.

2.3 Passivity theory

2.3.1 Dissipativity and Passivity

Lyapunov stability theory regards the energy stored in the system as a way to deduce statements over the stability of a certain equilibrium. In order to achieve this, it regards the states where the energy is minimal. However, the energy function can also provide information about a number of other properties of a system. One method to analyse the system behaviour is to observe how the stored energy changes over the time. This leads to the properties *dissipativity* and *passivity*.

Considering a time invariant system with n inputs and n outputs

$$\Sigma = \begin{cases} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x} \in \mathbb{R}^n \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{u}, \boldsymbol{y} \in \mathbb{R}^m \end{cases} \tag{2.11}$$

with $\boldsymbol{u} \in \mathcal{U} := \{\boldsymbol{u} \in \mathbb{R}^m |$ all future $\boldsymbol{u}$ are bounded$\}$, it can be defined as follows [SJK12, p. 27]:

Definition 9 (Dissipativity): Assume for a system defined by Eq. (2.11) exists a function $w : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ which is called the supply rate. Furthermore assume that the supply rate is locally integrable for every $\boldsymbol{u} \in \mathcal{U}$, that is $\int_{t_0}^{t_1} |w(\boldsymbol{u}(t), \boldsymbol{y}(t))| dt < \infty$ for all $t_0 \leq t_1$. Let $\mathcal{X} \subset \mathbb{R}^n$ contain the origin.

The system is called *dissipative* in $\mathcal{X}$ with the supply rate $w(\boldsymbol{u}, \boldsymbol{y})$ if there exists a positive definite storage function $S(\boldsymbol{x})$ with $S(\boldsymbol{0}) = \boldsymbol{0}$ such that for all $\boldsymbol{x} \in \mathcal{X}$

$$S(\boldsymbol{x}) \geq 0 \text{ and } S(\boldsymbol{x}(t_1)) - S(\boldsymbol{x}(t_0)) \leq \int_{t_0}^{t_1} w(\boldsymbol{u}(t), \boldsymbol{y}(t)) \; dt \tag{2.12}$$

for all $\boldsymbol{u} \in \mathcal{U}$ and all $t \in [t_0, t_1]$.

Definition 10 (Passivity): A system defined by Eq. (2.11) is called *passive*, if it is dissipative with the supply rate

$$w(\boldsymbol{u}, \boldsymbol{y}) = \boldsymbol{u}^\top \boldsymbol{y} \tag{2.13}$$

Corollary The formulation in Definition 9 is an integral representation of dissipativity. If $S(\boldsymbol{x})$ is differentiable, it can also be written in a differential form as

$$\dot{S}(\boldsymbol{x}) \leq w(\boldsymbol{u}(t), \boldsymbol{y}(t)). \tag{2.14}$$

For passivity, consequently, the relationship also holds. ∎

Remark 5: For physical systems, passivity is only a reformulation of energy balancing [BOS21]: The inner product $\boldsymbol{u}^\top \boldsymbol{y}$ can be interpreted as the power that flows over the system borders. Since energy can never be produced or destroyed, the stored energy in the system can either change according to the power flow over the system ports or it can be dissipated[1]. Accordingly, the derivative of the energy function has to be equal to $\boldsymbol{u}^\top \boldsymbol{y}$ (the power flow) or smaller (additional dissipation).

Remark 6: Passivity is a system property and not bound to an equilibrium. Passivity depends on the input-output combinations and therefore it is e.g. possible to say "the system is passive with respect to the interconnection structure" if the $\boldsymbol{d}$-$\boldsymbol{z}$ ports are meant which are introduced later in this chapter. In this work, additionally the expression "the system is passive with respect to a state $\boldsymbol{x}^*$" is used to express that the system is passive and the associated storage function which proves passivity is minimal at the state $\boldsymbol{x}^*$.

A special definition of passivity is output strictly passivity [Kha02, p. 231]:

Definition 11 (Output Strict Passivity): A system of the form Eq. (2.11) is called *output strictly passive*, if it has a storage function as in Definition 9 and the inequality

$$\dot{H}(\boldsymbol{x}) \leq \boldsymbol{u}^\top \boldsymbol{y} - \boldsymbol{y}^\top \boldsymbol{\rho}(\boldsymbol{y}) \tag{2.15}$$

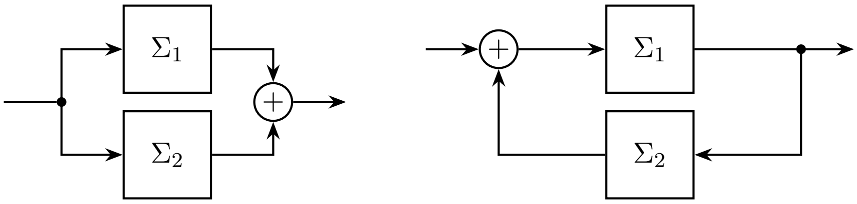
holds for all $\boldsymbol{u} \in \mathcal{U}$ and $\boldsymbol{y} \in \mathcal{Y}, \boldsymbol{y} \neq \boldsymbol{0}$ and with $\boldsymbol{y}^\top \boldsymbol{\rho}(\boldsymbol{y}) > 0$.

At the moment, this is only a more strict definition of passivity. It can be understood in such way that the system is "more passive" than other passive systems. Its usefulness becomes visible in connection with PI control for passive systems in Section 2.5.1.

The behaviour of passive systems during interconnection is an interesting property and one of the main reasons why passivity is so widely used [SJK12, p. 33]:

Proposition 1 (Interconnection of passive systems): Suppose two systems are passive and interconnected in a parallel or feedback interconnection as depicted in Figure 2.1. Then the resulting interconnected system is also passive. The storage function of the interconnected

[1] Strictly speaking, dissipation is also equal to a power flow over the system borders, only that this port is not explicitly modeled.



(a) Parallel interconnection (b) Feedback interconnection

Figure 2.1: Interconnection of two systems Σ_1 and Σ_2.

system can be found by adding the single storage functions $H_i(\boldsymbol{x}_i)$, $i = 1, \ldots, n$:

$$H_{\text{tot}}(\boldsymbol{x}) = \sum_i H_i(\boldsymbol{x}_i) \tag{2.16}$$

This makes passivity a very appealing property for the analysis of interconnected systems. In contrast to stability, passivity is preserved during interconnection. Properties of the whole interconnected network can be derived from its single components.

2.3.2 Passivity and Stability

As mentioned at the beginning of this section, passivity and Lyapunov stability both use energy functions to derive statements over a system. This strong shared basis leads to equally strong connections between the properties.

The first observation that can be made, is the following:

Corollary If a system is passive and the input $\boldsymbol{u}$ can be chosen $\boldsymbol{u} = \boldsymbol{0}$, then the inner product $\boldsymbol{u}^\top \boldsymbol{y}$ is also zero. In this case, from Definition 10, the definition of passivity, follows directly stability with respect to the minimum of the energy function. ■

This is a very useful fact and the main reason why passivity is such a useful feature. Its importance reveals itself in combination with the preservation feature of passivity (Proposition 1):

Remark 7: It is hard to find a direct stability proof for possibly nonlinear, interconnected systems with a variable structure. Every system has its own dynamic and without further information it cannot foreseen how the systems will interact.

But if every subsystem is passive, the resulting overall system will be passive too, assuming a structure in line with Proposition 1. If it is now possible to ensure that all inputs to the outside are zero as in Corollary 2, the equilibrium of the overall system will be stable.

This remark can serve as the fundamental motivation for many of the considerations made in this work. It means that two things are important for the controller design of interconnected systems:

The first one is to set the desired equilibrium individually asymptotically stable. The second is to shape every closed-loop subsystem passive. If the controllers are designed in a manner that ensures passivity of the closed loop, then the equilibrium of the interconnected system can also be set asymptotically stable.

Therefore the treatment of the subsystems will focus mainly on the passivity properties in the rest of the work. Passivity is sought, not as a necessity for the controller design, but as a property to ensure asymptotic stability of the DHN as a whole.

This approach has been used in many works in different practical applications. Examples are all kinds of energy networks as depicted in [Str+20; Mal; NFT19; LC21; Cuc+19; Men+17], among many others.

2.3.3 Passivity of Incremental Systems and Equilibrium-independent Passivity

In its original form, passivity is defined using an energy function. This one is usually equal to the Lyapunov function of the system and it is mostly minimal at *zero-state*. As passivity is used to ensure stability of an interconnected network, this can be a mayor drawback: In many control applications one is interested in operating the system around a *non-zero* equilibrium point. In these cases, passivity alone is not sufficient and more precise criteria are needed.

As to expect, solutions for this problem exist and two of them are presented here. The first one aims at a distinct equilibrium that is different from zero even if the original storage function is minimal at zero (or any other point). It is called *passivity of incremental systems*. The second one tries to show passivity with regard to any possible equilibrium and is called *equilibrium-independent passivity* (EIP).

Passivity of Incremental Systems Passivity of incremental systems has been defined for the first time in [Jay+07].

Definition 12 (Passivity of Incremental Systems): A system of the form

$$\Sigma := \begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \end{cases} \tag{2.17}$$

with $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u}, \boldsymbol{y} \in \mathbb{R}^m$, a constant input matrix $\boldsymbol{G}$ of full rank and $n \leq m$ is considered. Suppose $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ are locally Lipschitz and fix an equilibrium $\boldsymbol{x}^*$ in the set of possible equilibria

$$\boldsymbol{x}^* \in \mathcal{E} = \{\tilde{\boldsymbol{x}} \in \mathbb{R}^n | \boldsymbol{G}^\perp \boldsymbol{f}(\tilde{\boldsymbol{x}}) = \boldsymbol{0}\}. \tag{2.18}$$

where $\boldsymbol{G}^\perp$ is a full-rank left-annihilator, i.e. $\boldsymbol{G}^\perp \boldsymbol{G} = \boldsymbol{0}$. Define $\boldsymbol{u}^*$ and $\boldsymbol{y}^*$ as the constant inputs and outputs belonging to the equilibrium $\boldsymbol{x}^*$.

Use the incremental variables $\tilde{(\cdot)} = (\cdot) - (\cdot)^*$ and define the incremental model

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{u}^* + \boldsymbol{G}\tilde{\boldsymbol{u}} \\ \tilde{\boldsymbol{y}} &= \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{h}(\boldsymbol{x}^*). \end{aligned} \tag{2.19}$$

The system is called *incrementally passive* if the mapping $\tilde{\boldsymbol{u}} \to \tilde{\boldsymbol{y}}$ is passive.

Remark 8: The incremental model can be seen as the model that describes the error towards an equilibrium $\boldsymbol{x}^*$. Accordingly, incremental passivity follows as desired as the passivity with respect to $\boldsymbol{x}^*$. In order to show incremental passivity, it is frequently convenient to use the same storage function as used for the original system only with the incremental variable $\tilde{\boldsymbol{x}}$, i.e. $H(\tilde{\boldsymbol{x}}) = \tilde{H}(\tilde{\boldsymbol{x}})$.

Equilibrium-independent Passivity The other property to be mentioned is EIP. It can be given formally by the following two definitions [Hin11]: The first one gives some preliminary assumptions and definitions and the second one contains the actual passivity property.

> **Definition 13:** Assume for a system of the form (2.11) exists a nonempty set $\mathcal{U}^* \subseteq \mathcal{U}$ such that for every $\boldsymbol{u}^* \in \mathcal{U}^*$ there exists a unique $\boldsymbol{x}^* \in \mathcal{X}$ such that $\boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{u}^*) = \boldsymbol{0}$. Define a mapping
>
> $$k_x : \mathcal{U}^* \to \mathcal{X}$$
>
> such that $\boldsymbol{f}(k_x(\boldsymbol{u}^*), \boldsymbol{u}^*) = 0$ and assume it is continuous. Furthermore define another mapping
>
> $$k_y : \mathcal{U}^* \to \mathcal{Y}$$
>
> with $k_y(\boldsymbol{u}^*) := h(k_x(\boldsymbol{u}^*), \boldsymbol{u}^*)$ that is consequently also continuous.

This allows to ensure that only admissible inputs are used which result in a unique constant output.

> **Definition 14** (Equilibrium-independent passivity): Assume for a system of the form (2.11) exist $\boldsymbol{u}^* \in \mathcal{U}^*$, $\boldsymbol{x}^* = k_x(\boldsymbol{u}^*) \in \mathcal{X}$ and $\boldsymbol{y}^* = k_y(\boldsymbol{u}^*) \in \mathcal{Y}^*$ as given in Definition 13. The system is called *equilibrium-independent passive* (EIP) if for every $\boldsymbol{u}^* \in \mathcal{U}^*$ there exists a once-differentiable storage function $S_{\boldsymbol{u}^*} : \mathcal{X} \to \mathbb{R}$ such that $S_{\boldsymbol{u}^*}(\boldsymbol{x})$ is positive definite and
>
> $$\dot{S}^{\top}_{\boldsymbol{u}^*}(x, u) := \nabla_x S_{\boldsymbol{u}^*}(\boldsymbol{x}) \cdot \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \leq (\boldsymbol{u} - \boldsymbol{u}^*)^{\top}(\boldsymbol{y} - \boldsymbol{y}^*)$$
>
> for all $\boldsymbol{u} \in \mathcal{U}, \boldsymbol{x} \in \mathcal{X}$.

> **Remark 9:** Both EIP as well as incremental passivity are pretty similar system properties. The first and most important difference is that EIP is independent from the equilibrium and holds for every $\boldsymbol{u}^* \in \mathcal{U}^*$. Incremental passivity in contrast is limited per definition to one equilibrium: While the incremental system can be defined for all possible equilibria, not every one is necessarily passive. Therefore EIP is a more strict property that is fulfilled by less systems.

> **Remark 10:** When both properties are used for the stability analysis of a controller design (see Section 2.3.2), they can be applied in different use cases: A controlled subsystem of a network can be assumed to be passive only with respect to the desired equilibrium, given a suitable controller design. After all, the equilibrium is actively adjusted by the controller. Consequently, incremental passivity is the correct criterion here.
>
> An uncontrolled subsystem, on the other hand, cannot drive a desired equilibrium on its own. If it does adopt an equilibrium, this is instead determined by the connected systems. Consequently, the equilibrium is not known in advance. In order to be able to consider the system in the controller design anyway, EIP must be used that is passive with respect to every possible equilibrium.

2.4 Modeling of Physical Systems

After discussing quite a number of system properties, it has not yet been clarified how to determine the systems in the first place. The next section is dedicated to this question and it is shown how, based on the chosen model structure, already some properties of the model can be given.

There are quite a number of different modeling techniques that can depict systems and that represent them mathematically. Especially interesting for this work are methods which consider system energy. This provides a direct link to the desired passivity and stability properties discussed earlier in this chapter.

Besides, the used modeling techniques follow another aim: Observations have been made that these methods also provide a general view on system dynamics that is independent from the actual physical domain. Instead, systems from such different domains as mechanical, electrical or fluid domain have similar properties and can be modeled in one common framework [Wel79, p. 25].

This shall be used in this work and therefore bond graphs and the generalized electric circuit diagram are introduced in Section 2.4.1. Among other things, these are subject to certain constraints on the interconnection between each other which are illustrated in Section 2.4.2. Afterwards, Input-state-output Port-Hamiltonian Systems (ISO-PHS) are described in Section 2.4.3. These are used as the mathematical system models in this work.

2.4.1 Bond Graphs and the Generalized Electric Equivalent Circuit Diagram

Bond graph modeling is a systematic modeling technique that tries to abstract the modeled systems via energy ports. It is a broad field which is not presented here in the full length, but the interested reader may refer to [Wel79, ch. 8]. The following statements are based on the same source, see [Wel79, pp. 170-177].

In bond graph modeling, the system is separated in several subsystems that can either act as energy source, as energy storages or as dissipators. Additionally, there are transformers and gyrators which are not treated here. The systems are connected with so-called bonds which transport the energy in a power-conserving manner between them.

The effective power at a certain point of time can be represented by the product of two variables, the generalized effort e and the generalized flow f. Depending on the domain of the system, they can take different units and dimension. In electric circuits for example, the effort represents the voltage and the flow represents the current. In the hydraulic domain, the pressure can be depicted by the effort and the volume flow by the generalized flow.

Also sources, storages and dissipators take different forms depending on the physical domain. An ideal effort source corresponds to a voltage source in the electrical domain and to a pump in the hydraulic domain. In other words, a hydraulic source has the same effect on a hydraulic system as a voltage source has on an electric circuit. This approach can be continued, since capacitors, inductors and resistances also have a hydraulic respectively generalized equivalent: Capacitors correspond to pressure tanks, inductors to ideal hydraulic pipes and resistors to hydraulic resistors.

This reveals a possibility to depict dynamical hydraulic systems: Since the electrical components are equivalent to hydraulic components, it is possible to depict a hydraulic network as an electrical network and the other way round. If the hydraulic network[2] is displayed as electrical circuit, one speaks of a generalized electrical equivalent circuits (EECs) [Til96]. They have the same characteristics as the original system but provide a familiar environment for electrical engineers to work with. Therefore, this approach is adopted and the models used in this work are displayed as EECs.

It is clear that if (possibly nonlinear) EECs can be motivated through bond graphs, the exact opposite way is also possible. Fundamentally, both methods use the same philosophy of generalizing dynamic systems and are in a certain way equivalent. The statements in the following section are made for bond graphs but accordingly also hold for EECs.

[2] or any other network that can be displayed in generalized variables

2.4.2 Compatibility Contraints

One of the advantages of bond graphs is that everything that is within an energy port can be regarded as a black box from outside - not to be mistaken with black-box modeling - and the different black boxes can be interconnected easily. Though, some constraints have to be met so that the interconnection is well-defined. The following results describe this and are taken from [Wel79, pp. 170-177].

When different systems are interconnected at a junction, it is necessary to distinguish between effort junctions (also called 1-junctions) and flow junctions (also called 0-junctions). Effort junctions with n adjacent bonds can be described by the constitutive relationship

$$e_1 + e_2 + e_3 + \ldots + e_n = 0 \tag{2.20a}$$
$$f_1 = f_2 = f_3 = \ldots = f_n \tag{2.20b}$$

which relates for electrical domain to Kirchhoff's Voltage Law. For flow junctions the relation is

$$f_1 + f_2 + f_3 + \ldots + f_n = 0 \tag{2.21a}$$
$$e_1 = e_2 = e_3 = \ldots = e_n \tag{2.21b}$$

which is equal to Kirchhoff's Current Law for electric circuits.

These laws pose direct conditions on the compatibility of the connection, also called the *generalized compatibility constraint* (effort junctions) resp. the *generalized continuity constraint* (flow junctions): If a bond graph is to be calculated or simulated, it has to be brought in a mathematical form. In this case, either the effort or the flow variable can be regarded as a degree of freedom. The other variable is fixed by the system dynamics and the interaction between the systems. Determining the fixed and free variables is called the establishment of *mathematical causality* in literature.

To come back to compatibility constraints, this means the following: The bonds still need to fulfill the conditions at a junction Eqs. (2.20) and (2.21). In the case of effort junctions, all flows are the same (see Eq. (2.20b)) and accordingly one free flow variable fixes all the other flows at the junction. This means that also only one bond can have the flow as a fixed variable as two fixed flows could contradict each other. Instead, the other flows have to be mathematically variable so that they can take the same value all the time.

In contrast, the efforts at an effort junction all sum up to zero. There are consequently $n-1$ degrees of freedom. If one bond has a fixed effort as mentioned in the previous paragraph, it has thus a free flow. In this case, the other $n-1$ bonds can have a fixed effort variable exactly corresponding to the $n-1$ degrees of freedom and fulfilling the compatibility constraint. Analogous considerations apply to flow junctions and continuity constraints.

To summarize, the constraints determine how many flow variables and how many effort variables must be fixed at a junction so that the connection is mathematically well-defined. Within each bond, this also results in the other associated variable being free.

In order to use this with mathematical models - which according to previous considerations already have a fixed mathematical causality - only one additional thought is necessary: In a mathematical system on a signal level, bonds are not directly considered. But the input into a system can be seen as a free variable and the output (mostly) as a fixed one, which is defined by the input and the system dynamics. The compatibility constraints still apply. Instead of defining fixed and free variables at a bond, they now define the free inputs and the fixed outputs of a system. For the sake of clarity, the conditions are given in Table 2.1.

effort junctions		
	flow variables	effort variables
system input (variable)	$n-1$	1
system output (fixed)	1	$n-1$

(a)

flow junctions		
	flow variables	effort variables
system input (variable)	1	$n-1$
system output (fixed)	$n-1$	1

(b)

Table 2.1: Overview of how many variables of a type must be present for the junction to be well-defined.

2.4.3 Port-Hamiltonian Systems

In classic control theory, state-space models are commonly used. They attribute a state to the system in such a way that the system dynamics are completely defined by the current state and the current input. An energy-based modeling approach called Input-state-output port-Hamiltonian systems (ISO-PHS) now aims at combining these classic model forms with port-based modeling traditions as depicted in Section 2.4.1.

In order to explain the structure of ISO-PHS, the basics of port-Hamiltonian modeling are presented at the beginning of the section. The presentation is limited to the important aspects and the full range does not need to be explained for this work. In the case of further interest, the reader may refer to [Dui+09; VJ14; Sch17a]. In the following, some properties of PHS and connections to bond graph modeling are shown. Subsequently, the main outcome of this section is stated as the structure of ISO-PHS is defined and important characteristics are expounded.

Port-Hamiltonian systems Port-Hamiltonian systems (PHS) in general can take various shapes such as abstract geometrical forms [Dui+09, p. 64], coordinate-based representations [Dui+09, p. 84] or exactly the state-space based ISO-PHS [Dui+09, p. 69]. All of these consist of the four constituting elements [Dui+09, p. xii]

- power-conserving energy storages
- power-consuming dissipation elements
- a so-called "Dirac structure"
- and power-conserving ports to other systems or controllers.

The Dirac structure is a power-conserving interaction structure. Its definitions usually rely on geometric characteristics (ref. [VJ14, ch. 2.2]) that can be broken down to two things: The first one is (as mentioned) the power-preservation. The second one is that it fixes all free variables such that the system of storages, dissipators and interaction ports takes defined values depending from the input values. As [Dui+09, p. 56] mentions, flow and effort junctions are "prime examples" of the general concept of (constant) Dirac structures.

> **Remark 11:** Besides, flow and effort junctions can also be seen as Kirchhoff-Dirac structures, which are a bit stricter defined. This means that they additionally constrain the flows at the internal vertices to zero [VJ14, ch. 12.8]. Therefore, one the one hand, the power-conserving relationships are only defined at the interconnections of storages, dissipators and external ports and, on the other hand, they follow Kirchhoff's laws in a generalized manner. The thought is similar to the content of Section 2.4.2.

ISO-PHS The main objective of this section is to define the model class of ISO-PHS which is done in the following.

An ISO-PHS is a special form of PHSs that uses the stored energy in every storage as the system state. A system can be modeled as ISO-PHS under these necessary conditions [Dui+09, p. 69] and [VJ14, p. 53]:

1. The energy states $\boldsymbol{x}$ are not affected by algebraic constraints.
2. The external port variables can be divided into specific conjugated input-output pairs.
3. The resistive structure is linear with respect to the states and of input-output form.

That said, the formal definition can be given as follows based on [VJ14, p. 55], [Sch17a, pp. 113ff.], [Dui+09, pp. 69ff.] and [Mal, p. 40]:

Definition 15 (Input-state-output port-Hamiltonian system): An *input-state-output port-Hamiltonian system* (ISO-PHS) is a system of the form

$$\Sigma_{PHS} := \begin{cases} \dot{\boldsymbol{x}} &= [\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})]\nabla H(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{K}(\boldsymbol{x})\boldsymbol{d} \\ \boldsymbol{y} &= \boldsymbol{G}^\top(\boldsymbol{x})\nabla H(\boldsymbol{x}) \\ \boldsymbol{z} &= \boldsymbol{K}^\top(\boldsymbol{x})\nabla H(\boldsymbol{x}) \end{cases} \quad \boldsymbol{x} \in \mathcal{X} \tag{2.22}$$

with

$\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^n$	The state of the system
$\boldsymbol{u} \in \mathcal{U} \subseteq \mathbb{R}^m$	The control input
$\boldsymbol{y} \in \mathcal{Y} \subseteq \mathbb{R}^m$	The natural passive output
$\boldsymbol{d} \in \mathcal{D} \subseteq \mathbb{R}^d$	The uncontrolled disturbance and interconnection input
$\boldsymbol{z} \in \mathcal{Z} \subseteq \mathbb{R}^d$	The output conjugated to $\boldsymbol{d}$
$H(\boldsymbol{x})$	The positive definite energy storage function or also *Hamiltonian* of the system, where $H : \mathcal{X} \to \mathbb{R}$
$\nabla H(\boldsymbol{x}) = \frac{\partial H}{\partial \boldsymbol{x}}(\boldsymbol{x}) \in \mathbb{R}^n$	The gradient of $H(\boldsymbol{x})$ w.r.t. $\boldsymbol{x}$
$\boldsymbol{J}(\boldsymbol{x}) \in \mathbb{R}^{n \times n}$	The interconnection matrix
$\boldsymbol{R}(\boldsymbol{x}) \in \mathbb{R}^{n \times n}$	The dissipation matrix
$\boldsymbol{G}(\boldsymbol{x}) \in \mathbb{R}^{n \times m}$	The control input matrix
$\boldsymbol{K}(\boldsymbol{x}) \in \mathbb{R}^{n \times d}$	The disturbance and interconnection matrix

The matrix $\boldsymbol{R}(\boldsymbol{x})$ is symmetric and positive semidefinite. The matrix $\boldsymbol{J}(\boldsymbol{x})$ is negative semidefinite.

$$\boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{R}^\top(\boldsymbol{x}) \succcurlyeq 0 \tag{2.23a}$$

$$\boldsymbol{J}(\boldsymbol{x}) = -\boldsymbol{J}^\top(\boldsymbol{x}) \tag{2.23b}$$

The ports of the system are split up into the controllable inputs $\boldsymbol{u}$ and outputs $\boldsymbol{y}$ and the interaction inputs $\boldsymbol{d}$ and outputs $\boldsymbol{z}$. The first ones are connected to external actors and the controllable input is used to operate the system. The latter ones originate from the interconnection of one system with another and naturally cannot be controlled. This distinction is not obligatory, but it can help to keep an overview over the system and its ports.

The Hamiltonian function traces back to the 19th century, when it was used for the first time to describe system dynamics [Enc]. It describes the energy stored in the system and therefore can potentially be used as Lyapunov or storage function.

If the last assumption on the existence of ISO-PHS is dropped, this leads to the slightly broader definition of *ISO-PHS with nonlinear resistive structure* [Sch17a, p. 114]:

Definition 16 (ISO-PHS with nonlinear resistive structure): An ISO-PHS with nonlinear resistive structure can be given as

$$\Sigma_{PHS} := \begin{cases} \dot{\boldsymbol{x}} &= \boldsymbol{J}(\boldsymbol{x})\nabla H(\boldsymbol{x}) - \mathcal{R}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{K}(\boldsymbol{x})\boldsymbol{d} \\ \boldsymbol{y} &= \boldsymbol{G}^\top(\boldsymbol{x})\nabla H(\boldsymbol{x}) \\ \boldsymbol{z} &= \boldsymbol{K}^\top(\boldsymbol{x})\nabla H(\boldsymbol{x}) \end{cases} \qquad \boldsymbol{x} \in \mathcal{X} \tag{2.24}$$

where all elements are the same as in Definition 15 except of $\mathcal{R}(\boldsymbol{x})$ which is an arbitrary positive definite function with $\mathcal{R}(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}^n$.

Obviously, Definitions 15 and 16 are nearly identical with the only difference that the latter one allows for nonlinear resistances.

Passive outputs It is understood that the Hamiltonian as a function of the energy is the key element that connects port-based modeling with the state-space form. It can also be used as the Lyapunov function for proving stability or the storage function for proving passivity. This is a main advantage of ISO-PHS modeling: Storage functions (resp. Lyapunov functions) can take a quite general form and finding a suitable storage function may be not a trivial task. ISO-PHS modeling, however, delivers a suitable function directly by construction. Therefore, it enables an effortless application of passivity and stability theorems.

This consideration can even be extended [Mal, p. 39]:

Lemma Deriving the Hamiltonian of a system (2.22) w.r.t. the time brings

$$\begin{aligned} \dot{H}^\top(\boldsymbol{x}) &= \nabla H^\top(\boldsymbol{x})\dot{\boldsymbol{x}} \\ &= -\nabla H^\top(\boldsymbol{x})\boldsymbol{R}(\boldsymbol{x})\nabla H(\boldsymbol{x}) + \boldsymbol{y}^\top\boldsymbol{u} + \boldsymbol{z}^\top\boldsymbol{d} \leq \boldsymbol{y}^\top\boldsymbol{u} + \boldsymbol{z}^\top\boldsymbol{d}. \end{aligned} \tag{2.25}$$

Accordingly, any possible ISO-PHS is passive w.r.t. its input-output pairs $\boldsymbol{u}$ and $\boldsymbol{y}$ resp. $\boldsymbol{d}$ and $\boldsymbol{z}$. ∎

This is the reason why the output $\boldsymbol{y}$ is sometimes also called *natural passive output.* Note that other passive input-output pairs are also possible, but their existence is not a priori guaranteed. Still, all of them can be derived from the following parametrization [Zha+18].

Proposition 2: Consider a system with a PHS state-space equation as in Definition 15 and an arbitrary output function $\boldsymbol{y}_{\mathrm{wD}}(\boldsymbol{x}, \boldsymbol{u})$:

$$\Sigma := \begin{cases} \dot{\boldsymbol{x}} &= [\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})]\nabla H(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{K}(\boldsymbol{x})\boldsymbol{d} \\ \boldsymbol{y}_{\mathrm{wD}} &= \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{j}(\boldsymbol{x})\boldsymbol{u}. \end{cases} \tag{2.26}$$

The mapping $\boldsymbol{u} \to \boldsymbol{y}_{\mathrm{wD}}$ is passive if the following factorisations are possible:

$$\boldsymbol{h}(\boldsymbol{x}) = \left[\boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{\phi}^\top(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x})\right]^\top \nabla H(\boldsymbol{x}) \tag{2.27}$$

$$\boldsymbol{j}(\boldsymbol{x}) = \boldsymbol{\omega}^\top(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x}) + \boldsymbol{D}(\boldsymbol{x}) \tag{2.28}$$

with

- the arbitrary mapping $\boldsymbol{\omega} : \mathbb{R}^n \to \mathbb{R}^{q\times n}$
- the arbitrary mapping $\boldsymbol{D} : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ with $\boldsymbol{D}(\boldsymbol{x})$ skew-symmetric and
- the mapping $\boldsymbol{\phi} : \mathbb{R}^n \to \mathbb{R}^{q\times n}$ that is a factorization of the dissipation matrix

$$\boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{\phi}^\top(\boldsymbol{x})\boldsymbol{\phi}(\boldsymbol{x}) \tag{2.29}$$

with $q \in \mathbb{N}$ satisfying $q \geq \operatorname{rank}\{\boldsymbol{R}(\boldsymbol{x})\}$.

This observation can also be used in control as shown in Section 2.5.1.

Interconnection of PHS For general passive systems, interconnected passive systems stay passive according to the conditions in Proposition 1. These restrict the preservation of passivity to parallel and feedback interconnections. In the case of PHS, a more general statement can be made [Mal, p. 40]:

Proposition 3 (Interconnection of port-Hamiltonian systems): The interconnection of $k \in \mathbb{N}$ PHSs of the form (2.22) is considered. Each of them is indexed by $i = 1, \ldots, k$ with Hamiltonians $H_i(\boldsymbol{x}_i)$, states $\boldsymbol{x}_i \in \mathcal{X}_i$ and equilibria

$$\boldsymbol{x}_i^* = \arg\min_{\boldsymbol{x}_i} H_i(\boldsymbol{x}_i). \tag{2.30}$$

If the interconnection is a Dirac structure (i.e. a power-conserving interconnection), it can be represented by another PHS with the state and equilibrium vectors

$$\boldsymbol{x}_M := \begin{pmatrix} \boldsymbol{x}_1^\top & \ldots & \boldsymbol{x}_k^\top \end{pmatrix}^\top \tag{2.31}$$

$$\boldsymbol{x}_M^* := \begin{pmatrix} \boldsymbol{x}_1^{*\top} & \ldots & \boldsymbol{x}_k^{*\top} \end{pmatrix}^\top, \tag{2.32}$$

the Hamiltonian

$$H_M(\boldsymbol{x}_M) := H_1(\boldsymbol{x}_1) + \cdots + H_k(\boldsymbol{x}_k) \tag{2.33}$$

and the time derivative of the Hamiltonian

$$\dot{H}_M(\boldsymbol{x}_M) := \dot{H}_1(\boldsymbol{x}_1) + \cdots + \dot{H}_k(\boldsymbol{x}_k). \tag{2.34}$$

While the Theorem allows for arbitrary forms of interconnection instead of only parallel and feedback ones, it has also some restrictions that make it slightly less universal than one may think on the first view. The pitfalls of Proposition 3 are discussed in the following remarks.

Remark 12: If two or more equilibria are dependent of each other, this will change the general equilibrium of the connected system. The equilibria only stay unchanged if the interconnection does not add any dependencies. Put another way, all states that are independent of the interconnection will have equilibria unaffected by the interconnection.

Remark 13: A simple example for the phenomenon described in Remark 12 can be two capacitors next to each other.

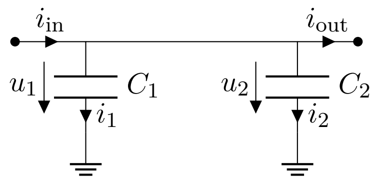


Figure 2.2: Electric circuit diagram of two capacitors in parallel.

Every individual one can be seen as a PHS with $k = 1, 2$, $g = 1$ and $R(x) = J(x) = 0$. Using the voltage u and the input to the system i yields

$$\begin{aligned} H(x_k) &= \frac{x_k^2}{2C_k} = \frac{(C_k u_k)^2}{2C_k} \\ \dot{x} &= C_k \dot{u}_k = i_k \\ y &= u. \end{aligned}$$

Accordingly, Proposition 3 holds and both systems together are passive. But from a dynamical perspective, there are not two systems acting to the outside and between each other. Rather, both capacitors act as one bigger capacitor with a capacity $C = C_1 + C_2$ and a voltage $u = u_1 = u_2$.

Although the capacitors were two separated and independent systems as long as they were not connected, now the two system states are dependent of each other and the connected system has only one degree of freedom. A counterexample where no additional dependency is added is the parallel interconnection of a capacitor with an inductance.

Remark 14: From a physical point of view, it is obvious that the different systems will affect each other in their dynamic behaviour. This is completely in line with the formulation of Proposition 3 which only states the preservation of passivity. It does not assure that undesirable behaviour appears, such as oscillations between different subsystems of the unwanted coupling of states. To ensure that this does not happen, it may be necessary to deploy in addition different methods such as LaSalle's invariance principle or to ensure that the interconnection happens in a certain way.

Even if the possible problems just described must be taken into account, Proposition 3 remains a useful tool for analyzing interconnected PHSs. For showing stability of interconnected systems, the general approach stays the same as described in Section 2.3.2. Only the argumentation is slightly changed with the use of Proposition 3.

2.5 Nonlinear Control

The main objective of this work is to design a stable control scheme for DHN. To achieve this goal, some control approaches are presented below.

Since the individual pumps are distributed throughout the DHN, it is costly to use a centralized control approach. In this case, communication would have to be installed between the various controllers which is inherently expensive. The cost comes from the difficult communication design partly due to the variable network structure[3] and partly because the communication is very time-critical. Instead, the control is to be implemented in a distributed manner, as also presented in Sections 1.4 and 2.3.2. In order to ensure stability nevertheless, the control is to be passivity-based. This means that the controlled system is passive with respect to the interaction ports and a stability analysis can be provided as described in the last section.

In the course of this work, two different control procedures are used. The first one is proportional-integral passivity-based control (PI-PBC). This is a method which is very easy to deploy since PI(D)-controllers are widely known and common industrial practice. It is presented explained in Section 2.5.1.

Unfortunately, PI-PBC is not universally applicable and imposes some rather strict conditions on the underlying model. In consequence, another control method has to be used. A very versatile approach is interconnection and damping assignment passivity-based control (IDA-PBC). This method can control nonlinear input-affine systems and gives the controlled system the structure of a PHS (consequently also ensuring passivity). Its robustness can be increased by additional Integral Action (IA). Although IDA-PBC has a very general form, some drawbacks have been observed during the creation of this work. Both are included in this work: The method itself is presented in Section 2.5.2 and the observed drawbacks, since they are better understandable with knowledge of the models, are given in Section 3.2.4.

2.5.1 Proportional Integral Passivity-based Control

PI or PID control has proven to be one of the most successful and versatile control strategies ever developed and is widely used in practice. In fact, about 90% of all existing control loops use PID control [Kno06]. Its advantages include the complete rejection of disturbances, its robustness and intuitiveness. Additionally, PID controllers often allow the manual tunig of the parameters by an operator in a certain range [Kno06; LAC06]. For these reasons, there are cost-efficient standard components on the market that can be easily and quickly installed in any system. This further illustrates the importance and appeal of PID controllers.

Originally, PID controllers were developed for linear control theory. But as a consequence of the previous paragraph, the feasibility of an usage in nonlinear systems is highly desirable. This is in fact possible and the applicability is partly based on one property:

Lemma Every PI controller of the form

$$\dot{\xi} = -\tilde{y} \tag{2.35a}$$
$$u = K_{\mathrm{I}}\xi - K_{\mathrm{P}}\tilde{y} \tag{2.35b}$$

with $K_{\mathrm{I}}, K_{\mathrm{P}} > 0$ is output strictly passive with respect to the map $\tilde{\boldsymbol{y}} \to \boldsymbol{u}$. ∎

Proof Adapted from [Jay+07]: With the storage function

$$H_{\mathrm{PI}}(\xi) = \frac{K}{2}\xi^2$$

and without loss of generality choosing $K = \frac{K_{\mathrm{I}}}{K_{\mathrm{P}}}$ follows

$$\dot{H}_{\mathrm{PI}}(\xi) = \nabla H_{\mathrm{PI}}^{\top}(\xi)\dot{\xi} = K\xi(-u) = K\left(\frac{1}{K_{\mathrm{I}}}u + \frac{K_{\mathrm{P}}}{K_{\mathrm{I}}}\tilde{y}\right)(-u) = u\tilde{y} - \frac{1}{K_{\mathrm{P}}}u^2$$ ∎

[3] The plug-in and plug-out of the individual customers changes the structure frequently.

Remark 15: Note that the input is $\tilde{y}$ and the output is u, exactly opposite to the previously presented systems. In feedback, the output u of the controller leads to the input u of the system and the other way around with y which explains the naming convention, see Figure 2.3.

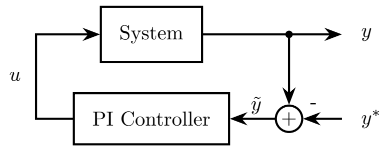


Figure 2.3: A PI controller in a closed-loop system.

In [Jay+07], this fact is used to show an interesting property of PI controllers:

Proposition 4 (PI Control for Nonlinear RLC Circuits): Every nonlinear RLC circuit can be written in the form of a PHS with nonlinear resistive structure (16). If

- the inductors and capacitors are passive and their energy functions are twice continuously differentiable and strictly convex,
- the resistors are passive and their characteristic functions are monotone non-decreasing and
- there is a equilibrium point $\boldsymbol{x}^*$ with constant input $\boldsymbol{u}^*$ and constant output $\boldsymbol{y}^*$ that is contained in the set

$$\boldsymbol{x}^* \in \mathcal{E} := \{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{G}^\perp \boldsymbol{f}(\boldsymbol{x})\} \tag{2.36}$$

  with $\boldsymbol{G}^\perp \boldsymbol{G} = \boldsymbol{0}$

then the PI controller

$$\begin{aligned} \dot{\xi} &= -\tilde{y} \\ u &= K_\mathrm{I}\xi - K_\mathrm{P}\tilde{y} \end{aligned} \tag{2.37}$$

ensures that the state trajectories of the system $\boldsymbol{x}$ and of the controller ξ are bounded and $\lim_{t\to\infty} \|\tilde{\boldsymbol{y}}(t)\| = \lim_{t\to\infty} \|\boldsymbol{y}(t) - \boldsymbol{y}^*\| \to \boldsymbol{0}$.

In fact, all strictly passive controllers can control this type of systems [Jay+07]. PI controllers again have two advantages: Firstly, they are a very convenient implementation of the bigger number of possible controls. But more important is the following lemma:

Lemma The closed loop system of a PHS with nonlinear resistive structure and a PI controller is passive with respect to the interaction ports, i.e. connections to other systems. ■

Proof A PID controller in feedback with a PHS with nonlinear resistive structure can again be written as a PHS with nonlinear resistive structure [Sch17a, p.153]. In consequence, the closed-loop system is passive with respect to the interaction ports. ■

This allows a passivity-based modular stability analysis in interconnected systems as previously discussed.

Remark 16: As discussed in Section 2.4, all of the models that are treated in this work can be written as nonlinear PHSs. Nevertheless, there is a limitation: As Definitions 15 and 16 show, in PHSs not only the states are fixed by the Hamiltonian, but also the outputs are fixed by

$\boldsymbol{G}(\boldsymbol{x})$ and $H(\boldsymbol{x})$. If the outputs are not the variables that are desired to control, a PI control law is not necessarily applicable. A solution can be to look for other passive input-output maps in the system as described in Proposition 2. Otherwise, another control method is necessary.

Remark 17: Recently, some other passivity-based control methods have been proposed that use PI and PID controllers. It is important not to confuse them because despite of a shared controller structure, they follow a different way of thinking about the control. PI-PBC according to [Jay+07] and as presented uses the strict passivity of any suitable PI-controller and the passivity of the model. The passivity remains according to Proposition 1 and the method follows rather a Control by Interconnection (CbI) approach.

PI(D)-PBC has been recently proposed in [Zha+18] and the following works [BON18; BOS21]. Here, the PI(D) controller is seen as a state-controller and the input in the controller is not necessarily exclusively the controlled variable. This means for the control that it is not sufficient only to measure the controlled variable but under circumstances a state-observer has to be deployed. On the upside, PID-PBC has a broader application range since the control of the states of the system is possible.

2.5.2 Interconnection and Damping Assignment Passivity-Based Control and Additional Integral Action

As described in Remark 16, PI-PBC is not every time applicable. Therefore another control method has to be used and in this work, IDA-PBC with additional IA has been chosen. The first step is to design an IDA-PBC controller that by itself can already control the system as desired. Subsequently, this control law is extended and made more robust by an additional IA. As both control methods are rather sophisticated, the single design steps of the whole process are given at the end in a summary.

IDA-PBC

IDA-PBC is a very flexible control method and can control systems of arbitrary structure. The basic approach is that the closed-loop system with a not yet known control structure should be the same as a desired PHS. IDA-PBC can be given as follows, combining results from [OGC04; BIW91]:

Proposition 5 (Interconnection and Damping Assignment Passivity-Based Control): Consider an nonlinear, input-affine system of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{K}(\boldsymbol{x})\boldsymbol{d} \tag{2.38}$$

with $\boldsymbol{d} = \boldsymbol{0}$ that is *feed back equivalent* to a passive system. Assume there are matrices

$$\boldsymbol{J}_d^\top(\boldsymbol{x}) = -\boldsymbol{J}_d(\boldsymbol{x}) \tag{2.39a}$$
$$\boldsymbol{R}_d^\top(\boldsymbol{x}) = \boldsymbol{R}_d(\boldsymbol{x}) \succcurlyeq 0, \tag{2.39b}$$

a full-rank left annihilator $\boldsymbol{G}^\perp(\boldsymbol{x})$ of $\boldsymbol{G}(\boldsymbol{x})$, i.e. $\boldsymbol{G}^\perp(\boldsymbol{x})\boldsymbol{G}(\boldsymbol{x}) = 0$ and a desired Hamiltonian $H_d(\boldsymbol{x})$ such that

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} H_d(\boldsymbol{x}), \tag{2.40}$$

i.e. the Hamiltonian is minimal at the desired equilibrium $\boldsymbol{x}^* \in \mathcal{X} \subseteq \mathbb{R}^n$ that is to be stabilised. Furthermore assume that the matching equation (ME), which is a PDE of the form

$$\boldsymbol{G}^{\perp}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{G}^{\perp}(\boldsymbol{x})[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})]\nabla H_d(\boldsymbol{x}), \tag{2.41}$$

is fulfilled for all $\boldsymbol{x}$. Then, the control law for the system takes the form

$$\boldsymbol{\beta}(\boldsymbol{x}) = \boldsymbol{G}^{+}(\boldsymbol{x})\Big[[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})]\nabla H_d(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x})\Big] \tag{2.42}$$

with

$$\boldsymbol{G}^{+}(\boldsymbol{x}) = \Big[\boldsymbol{G}^{\top}(\boldsymbol{x})\boldsymbol{G}(\boldsymbol{x})\Big]^{-1}\boldsymbol{G}^{\top}(\boldsymbol{x}) \tag{2.43}$$

the pseudo inverse of $\boldsymbol{G}(\boldsymbol{x})$. In this case, the closed-loop system with $\boldsymbol{d} = \boldsymbol{0}$ takes the port-Hamiltonian form

$$\dot{\boldsymbol{x}} = [\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})]\nabla H_d(\boldsymbol{x}) \tag{2.44}$$

with $\boldsymbol{x}^*$ a (locally) stable equilibrium. It will be asymptotically stable if in addition $\boldsymbol{x}^*$ is an isolated minimum of $H_d(\boldsymbol{x})$ and the largest invariant set under the closed-loop dynamics (Eq. (2.44) contained in

$$\mathcal{E} = \Big\{\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^n | \nabla H_d^{\top}(\boldsymbol{x})\boldsymbol{R}_d(\boldsymbol{x})\nabla H_d(\boldsymbol{x}) = 0\Big\} \tag{2.45}$$

equals $\{\boldsymbol{x}^*\}$. An estimate of its domain of attraction is given by the largest bounded level set

$$\{\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^n \mid H_d(\boldsymbol{x}) \leq c\}. \tag{2.46}$$

Remark 18: Since PHS are inherently passive, the feedback-equivalence (ref. [BIW91]) is always given as long as IDA-PBC is used to control PHSs.

The strength of IDA-PBC is that it allows for many degrees of freedom. $\boldsymbol{J}_d(\boldsymbol{x})$, $\boldsymbol{R}_d(\boldsymbol{x})$ and $\nabla H_d(\boldsymbol{x})$ can be freely chosen with the only constraint that [OGC04]

- the ME (2.41) has to be fulfilled and
- $\boldsymbol{R}_d(\boldsymbol{x})$ and $\boldsymbol{J}_d(\boldsymbol{x})$ have to be symmetric and skew-symmetric, respectively.

Assuring the solvability of the ME is the most difficult task here. On the downside of course, it can become difficult to oversee how certain design choices affect the resulting model and control law.

In general, there are three different approaches to solve the ME [OGC04]:

Non-Parameterized IDA: In this case, the interconnection ($\boldsymbol{J}_d(\boldsymbol{x})$) and damping ($\boldsymbol{R}_d(\boldsymbol{x})$) matrices are fixed as well as the additional degree of freedom $\boldsymbol{G}^{\perp}(\boldsymbol{x})$. The ME results as a PDE that has to be solved for $H_d(\boldsymbol{x})$ and among the possible solutions the one is chosen that fulfils Eq. (2.40).

Algebraic IDA: Starting from the opposite direction and choosing a desired energy function is another option. In this case, the ME becomes simply an algebraic equation in $\boldsymbol{J}_d(\boldsymbol{x})$, $\boldsymbol{R}_d(\boldsymbol{x})$ and $\boldsymbol{G}^{\perp}(\boldsymbol{x})$.

Parameterized IDA: The last option is not to choose $H_d(\boldsymbol{x})$ completely, but only its function class. Subsequently, the ME yields a PDE that must be solved. This can be desirable e.g. for mechanical systems.

Among these three options, non-parametrized IDA is the most universal approach since the interconnection and damping matrices can be left in a parametrized form at the beginning. In this case, however, additional conditions on the parameters have to be found that ensure the fulfillment of Eqs. (2.39) and (2.41). The easier approach therefore is algebraic IDA that is dependent of a proper choice of the Hamiltonian.

Additional IA

After the design of a IDA-PBC controller, it can be desirable to add another control layer: The integral action. This one is used to compensate the effects coming from measurement noise, disturbance inputs or dynamics that are not included in the model. In order to explain the procedure, some definitions have to be made [DJ09]:

Definition 17 (Relative Degree of States): Consider a PHS system as given in Definition 15. If the input $\boldsymbol{u}$ does not act on all of the states, the system can be written in a different way: With setting the interconnection inputs to zero $\boldsymbol{d} = \boldsymbol{0}$ and omitting the dependencies of $\boldsymbol{x}$ to simplify the notation, the new form follows as

$$\begin{aligned} \begin{pmatrix} \dot{\boldsymbol{x}}_i \\ \dot{\boldsymbol{x}}_h \end{pmatrix} &= \begin{pmatrix} \boldsymbol{J}_i - \boldsymbol{R}_i & \boldsymbol{J}_{ih} - \boldsymbol{R}_{ih} \\ \boldsymbol{J}_{hi} - \boldsymbol{R}_{hi} & \boldsymbol{J}_h - \boldsymbol{R}_h \end{pmatrix} \begin{pmatrix} \partial H/\partial \boldsymbol{x}_i \\ \partial H/\partial \boldsymbol{x}_h \end{pmatrix} + \begin{pmatrix} \boldsymbol{G}_i \\ \boldsymbol{0} \end{pmatrix} \boldsymbol{u} \\ \boldsymbol{y} &= \begin{pmatrix} \boldsymbol{G}_i^\top \\ \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \partial H/\partial \boldsymbol{x}_i \\ \partial H/\partial \boldsymbol{x}_h \end{pmatrix}. \end{aligned} \tag{2.47}$$

The n_i directly actuated states $\boldsymbol{x}_i$ are called *relative-degree-one* (RD1) states and the $n_h = n - n_i$ remaining states $\boldsymbol{x}_h$ are called the *higher-relative-degree* (HRD) states.

The goal is to add IA and at the same time to preserve the PHS structure as this ensures the passivity of the controlled system. While the design of an IA is a rather trivial task for the RD1 states, it becomes a bit more complicated for HRD states and can be done as follows [DJ09]:

Proposition 6 (Additional Integral Action): Consider a system of the form (2.47). Assuming that a suitable control exists and using the state transformations $\boldsymbol{x}_h = \boldsymbol{s}_h$ and $\Psi : \boldsymbol{x}_i, \boldsymbol{x}_h, \boldsymbol{s}_e \rightarrow \boldsymbol{s}_i$, the controlled system can be written as the *extended target PHS* with IA on the HRD states

$$\begin{pmatrix} \dot{\boldsymbol{s}}_i \\ \dot{\boldsymbol{s}}_h \\ \dot{\boldsymbol{s}}_e \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_i - \boldsymbol{R}_i & \boldsymbol{J}_{ih} - \boldsymbol{R}_{ih} & \boldsymbol{0} \\ \boldsymbol{J}_{hi} - \boldsymbol{R}_{hi} & \boldsymbol{J}_h - \boldsymbol{R}_h & \boldsymbol{K}_I \\ \boldsymbol{0} & \boldsymbol{K}_I^\top & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \partial H_{d\boldsymbol{s}}/\partial \boldsymbol{s}_i \\ \partial H_{d\boldsymbol{s}}/\partial \boldsymbol{s}_h \\ \partial H_{d\boldsymbol{s}}/\partial \boldsymbol{s}_e \end{pmatrix} \tag{2.48}$$

using the extended Hamiltonian

$$H_{d\boldsymbol{s}}(\boldsymbol{s}_i, \boldsymbol{s}_h, \boldsymbol{s}_e) = H_d(\boldsymbol{s}_i, \boldsymbol{s}_h) + \frac{1}{2} \boldsymbol{s}_e^\top K^{-1} \boldsymbol{s}_e \tag{2.49}$$

and $K^{-1} > 0$.

For the extended target PHS and the control to exist, the conditions on the state transformation

$$(\boldsymbol{J}_{hi} - \boldsymbol{R}_{hi}) \frac{\partial H_d}{\partial \boldsymbol{x}_i} + (\boldsymbol{J}_h - \boldsymbol{R}_h) \frac{\partial H_d}{\partial \boldsymbol{x}_h} = (\boldsymbol{J}_{hi} - \boldsymbol{R}_{hi}) \frac{\partial H_{d\boldsymbol{s}}}{\partial \boldsymbol{s}_i} + (\boldsymbol{J}_h - \boldsymbol{R}_h) \frac{\partial H_{d\boldsymbol{s}}}{\partial \boldsymbol{s}_h} - \boldsymbol{K}_I \frac{\partial H_{d\boldsymbol{s}}}{\partial \boldsymbol{s}_e} \tag{2.50}$$

have to be met and Ψ has to be invertible. Alternatively, it can be shown that $(\partial^\top \Psi / \partial x_i)^{-1}$

exists. The control law $\boldsymbol{v}$ must satisfy

$$(\boldsymbol{J}_i - \boldsymbol{R}_i)\frac{\partial H_{ds}}{\partial \boldsymbol{s}_i} + (\boldsymbol{J}_{ih} - \boldsymbol{R}_{ih})\frac{\partial H_{ds}}{\partial \boldsymbol{s}_h} = \frac{\partial^\top \Psi}{\partial \boldsymbol{x}_i}\left[(\boldsymbol{J}_i - \boldsymbol{R}_i)\frac{\partial H_d}{\partial \boldsymbol{x}_i} + (\boldsymbol{J}_{ih} - \boldsymbol{R}_{ih})\frac{\partial H_d}{\partial \boldsymbol{x}_h} + \boldsymbol{G}_i(\boldsymbol{x})\boldsymbol{v}\right] + \frac{\partial^\top \Psi}{\partial \boldsymbol{x}_h}\dot{\boldsymbol{x}}_h + \frac{\partial^\top \Psi}{\partial \boldsymbol{s}_e}\dot{\boldsymbol{s}}_e \tag{2.51}$$

The resulting system can be regarded as follows: The structure of the original system has been preserved and extended by an integral action. The $\boldsymbol{x}_i$ states are changed to the $\boldsymbol{s}_i$ states while the $\boldsymbol{x}_h$ states are equal to the $\boldsymbol{s}_h$ states and the $\boldsymbol{s}_e$ states - the states of the integrals - are added.

Control Design Steps

Even though all necessary conditions and equations are given, IDA-PBC and IA both can seem somewhat unclear at first glance. Therefore, the single design steps are given here in a structured way.

Proposition 7 (Design Steps for IDA-PBC): Given a system of the form (2.38), the algebraic IDA can be implemented in line with Proposition 5 as follows:

Step 1: Select a suitable Hamiltonian $H_d(\boldsymbol{x})$. If a later step reveals that the ME cannot be fulfilled with the chosen function, go back to this step.

Step 2: Ensure that the parametrized matrices $J_d(\boldsymbol{x})$ and $\boldsymbol{R}_d(\boldsymbol{x})$ are skew-symmetric resp. symmetric and positive definite.

Step 3: Choose the parametrized $\boldsymbol{G}(\boldsymbol{x})^\perp$ so that $\boldsymbol{G}(\boldsymbol{x})^\perp \boldsymbol{G}(\boldsymbol{x}) = 0$ is fulfilled.

Step 4: Use the previous results to solve the ME (2.41).

Step 5: Calculate the control law (2.42).

Afterwards, integral action is added as follows:

Proposition 8 (Design Steps for Additional IA): The additional IA is deployed on the resulting PHS from Step 5 in accordance with Proposition 6 as given:

Step 6: Formulate the desired extended target PHS.

Step 7: Use Eq. (2.50) to solve for $\boldsymbol{s}_i$. The resulting correlation is the function for the state transformation Ψ. Check whether Ψ is invertible.

Step 8: Solve Eq. (2.51) for the control law $\boldsymbol{v}$

After the design of the individual control methods, it is advisable to check whether the desired PHS structure is achieved.

2.6 Conclusion

This chapter has given a broad overview over the tools of the trade in this work. The final goal is to design controllers for future DHN and to prove the stability of a desired equilibrium.

Since these networks will be interconnected and of variable structure, it is not possible to show the stability directly. Instead, this can be reached by using the various passivity features that have been discussed. They remain intact during linking and therefore the passivity of a whole network can be shown.

If such a result is present, it can be used to prove the stability: Passivity in connection with an input equal to zero implies stability of an equilibrium. As an extension, LaSalle's invariance principle has been treated. This can even show asymptotic stability by using general stability and exploiting system dynamics.

The rest of the chapter was dedicated to the question how passivity can be shown. This can be done via the definition, but that requires a suitable storage function. The latter can be difficult to find and therefore it was shown how to simplify the passivity analysis. This can be done in two different ways, via modeling and via control.

In the case of modeling, this works as follows: The modeling as PHS is based on Hamiltonian dynamics and thus on energy functions. Therefore, the modeling directly yields an energy function suitable for the passivity and stability analysis. Moreover, the interconnections of PHSs yield PHSs again, which further confirms the preservation of passivity in this particular system class.

While PHSs already have passivity by themselves, as uncontrolled systems they do not converge to a desired equilibrium but to any. In order to drive the individual systems to a certain operating point, a suitable control system must therefore be designed. This should additionally make the closed-loop system passive, so that a modular stability analysis remains possible. In order to achieve this, PI-PBC and IDA-PBC with additional IA were introduced. Both can control PHSs and fulfill the requirements just mentioned.

The rest of the work will use these principles and methods in order to design the controllers and conduct the stability analysis. The first step is to derive appropriate models which is carried out in the next chapter. Additionally, some model properties will be shown that give insight in how the control should look like. Afterwards, appropriate controllers are designed in Chapter 4 and the stability analysis is given.

Chapter 3

Modeling and System Analysis

This chapter is dedicated to modeling typical 4GDHNs and 5GDHNs. The modeling objectives are outlined and necessary assumptions are presented. In the following, a short overview over the subsystems is given and DHNs are described formally. The individual subsystems are modeled under consideration of relevant dynamics and afterwards, the systems are analysed and first derivations for the control design are made.

3.1 Modeling

When modeling systems, different requirements for the model may contradict each other. On the one hand, the model should represent the real system in a sufficient way. It should therefore reduce the amount of information but keep the important aspects. These principles are also known as mapping and reduction. On the other hand, the model should be as simple as possible in order to allow an uncomplicated mathematical handling and simple control laws. This is known as pragmatism.

These three modeling principles, also known as the General Model Theory by Herbert Stachowiak [Sta73], lead to the insight that every model has a purpose and the modeling should be appropriate to this purpose. In modeling theory, it has therefore proven to be successful to distinguish between dynamic models, also called design models, and simulation models [Wel79, pp. 1ff.].

Design models provide a rather simple mathematical description and are used for system analysis and control design. They use sufficiently good enough approximations and provide information on a fundamental level. In return, they are easy to handle. Simulation models, in contrast, target on allowing an description as accurate as possible. They show the system in detail and can become very complex.

Since the motivation for this work is to develop controllers for future DHNs, the models are kept as simple as possible. They are developed for an easy mathematical handling and can be regarded as design models.

3.1.1 Assumptions

During the modeling, several simplifying assumption are made. An overview over the important assumptions is given here and an explanation can be found in the following sections at the suitable place:

- A1 The water is assumed to be incompressible.
- A2 The pipes and pumps have no elasticity. The heat exchanger, instead, has.
- A3 It is possible to calculate with the flow rate and pressure in a pipe averaged over the profile [Whi11, p. 370].

A4 The timescales of the thermal and the hydraulic domain can be treated separately.

A5 The whole DHN is on the same altitude. If otherwise, constant pressures have to be added which do not change the results, but clutter the notation.

A6 The desired steady state x^* to be set exists.

A7 The network structure satisfies the conditions in Proposition 19.

3.1.2 Components and Subsystems

Mathematical modeling of DHNs and its components is necessary to subsequently analyze and control them. First of all, however, they have to be described and defined in general to give an overview of the tasks and properties of the components.

It is helpful to see DHNs as networks of several different elements which are connected at clearly defined nodes. These elements are

- **Pipes** which establish the connection between the other subsystems.
- **Valves** which control the flow of fluids by opening and closing completely or in parts.
- **Pumps and booster pumps** which create the flow in a pipe system and a pressure increase between their input and output. Booster pumps are a special kind of pumps which are not placed at a household or a producer but in the middle of the network.
- **Pressure control devices**, also called pressure dictation devices, which define the absolute pressure in relationship to the ambience pressure. This stands in contrast to the aforementioned pumps that only can influence relative pressure differences and the volume flow.
- **Heat exchangers** which pose an intermediary interface between the DHN and the house substation [Nus20, pp. 86ff.].

In order to be able to control the different elements, it is useful to group them into subsystems. These subsystems are

- the **pipes**,
- the **producer substations** and
- the **consumer substations**.

The structure of producer and the consumer substations is relatively similar and they differ only with regard to the device for pressure control. These devices can be either pumps or gas vessels [Nus20, p. 54]. Pumps are active elements and they can set any desired pressure in the pipe. Gas vessels in contrary are passive and need to be filled correctly during the initial operation. Afterwards, they can only react passively on the changing conditions in the network.

Albeit the pumps offer a bigger flexibility than the gas vessels, both are used for the reason that gas vessels are much cheaper. Therefore, pumps are more often installed at bigger facilities, e.g. producers, while it is for smaller substations, e.g. consumer substations, more efficient to use gas vessels [Ref, p. 8]. The actual use of the substation may be interchangeable, but in order to achieve a precise designation, customer and producer substations are defined as follows:

Definition 18: Substations are the installations in a DHN that transmit the heat into or out of the network. They consist of a pump, a valve, a heat exchanger and a mechanism for pressure control. In customer substations, also called consumer substations, the pressure control is implemented via a pressure expansion vessels as seen in Figure 3.8. Producer substations are the substations with a pressure dictation pump and a pressure control as depicted in Figure 3.7.

This DHN as a whole, the pipes and this two subsystems are described and modeled in detail in the following sections. First of all, some remarks are made on where oscillations originate from in Section 3.1.3. They allow to decide which properties need to be represented in the developed models. Next, the interconnected DHN is considered in Section 3.1.4 followed by a section about the pipes in Section 3.1.5. The single elements of the producer and consumer substations are regarded in Section 3.1.6. Lastly, the producer substation and the consumer substation are modeled in Section 3.1.7 resp. Section 3.1.8.

3.1.3 Oscillations in District Heating Networks

During modeling, a simplification of the models has to be made in order to obtain the desired design models. This simplification has to be in accordance with the aims of the work. One of the main goals is to diminish the influence of oscillations in the pipe network and create a stable hydraulic state. Indeed, not all of the components have the same influence on oscillations and therefore a selection can be made [BT17]. Heat exchangers for example have a high elasticity and therefore are important. Valves and pipes in contrast have an insignificant influence and elastic parts do not need to be modeled. Equal considerations can be made for the pumps which are commonly rather massive builds.

Water per se can be regarded as incompressible, but only under the condition that the system is well ventilated [BT17]: It can happen that air comes into the system and air (as a gas) has a strong influence on the elasticity. Therefore, the maintainers of DHNs take care to ventilate the systems and keep the amount of air in the pipes as low as possible. If the water is assumed as incompressible, another effect is neglected which is known as water hammering. It appears when valves are opened or closed and a pressure stroke spreads in the network [Lei13, p. 1]. In this work, water hammering can be disregarded since passive dampers in DHNs exist that handle this problem [DPK09].

Due to the requested simplicity of the models, the modeling of elasticities in this work is limited to the heat exchangers since they have the biggest impact. The elasticities of the remaining parts is neglected (assumption A2) and the water is assumed to be incompressible (assumption A1).

In the following, the actual modeling is presented. This starts with a section about hydraulic networks since DHNs have this particular structure.

3.1.4 Hydraulic Networks

A formal modeling of DHNs as a whole can be done as hydraulic network, a type of networks which can be modeled as directed graph. The subsystems are represented by the edges and the vertices correspond to the connection of the pipes, also called nodes in the last section. Depending on the modeling, they can either contain fluid reservoirs or be a simple connection without storages.

While [VJ14, p. 126] calls both types of pipe networks hydraulic networks, in the proceeding a slightly more precise definition will be used:

Definition 19: The pipe networks with reservoirs will be called *flow networks* while networks without them will be denoted as *hydraulic network*.

This definitions are in line with literature, see [Jen12; TSD17; Vie]. In the case of DHNs, no reservoirs exist and thereby they can be seen as hydraulic networks. Under this knowledge, DHNs can be defined as in the literature [Str+; STD17; TSD17] and as presented in the following definition.

Definition 20: A district heating network (DHN) can be given as a weakly connected digraph $\mathfrak{G} = (\mathfrak{E}, \mathfrak{V})$ with the set of vertices $\mathfrak{V} = \{1, \dots, n_v\}$ and the set of directed edges $\mathfrak{E} = \{1, \dots, n_e\}$. Since the graph is biconnected and therefore possibly more edges than vertices can exist, it follows $n_e \geq n_v$. The topology is represented by the incidence matrix $B \in \mathbb{R}^{n_v \times n_e}$ with the edges $b_{ik} \in \{-1, 1\}$ and

$$b_{ik} = \begin{cases} +1 & \text{if the edge } b \text{ points from node } i \text{ to node } k \\ -1 & \text{if the edge } b \text{ points from node } k \text{ to node } i. \\ 0 & \text{if no connection exists.} \end{cases}$$

By combining the different network components in a meaningful way and using Definition 18, three different subsystems can be identified: Producer substations, consumer substations and pipes. In order to give a better overview over the network, the edges $\mathfrak{E}$ are partitioned according to these subsystems in three disjoint sets: $\mathfrak{D} = \{1, \dots, D\}$ represents the DGUs resp. producers, $\mathfrak{L} = \{D+1, \dots, D+L\}$ the consumer substations (loads) and $\mathfrak{P} = \{D+L+1, \dots, n_e\}$ the pipes.

Regarding the structure of a DHN, two observations can be made: Firstly, two substations are never connected directly to each other. Instead, they are always connected via pipes.

Secondly, the connection of components happens without a loss of energy. Additionally, a DHN is always closed and the water stays within the pipe network. This means that the generalized version of Kirchhoff's current law Eq. (2.20a) is applicable at every node [Str+21]. The hydraulic network therefore corresponds to a Kirchhoff-Dirac structure [VJ14, p. 126].

3.1.5 Pipes

The previous section describes DHNs mathematically as interconnected systems consisting of different subsystems. These subsystems itself are in contrast not yet modeled which is done in the following.

The fundamentals for this modeling lie in the field of fluid dynamics. This is a broad field of science and even presenting only the essentials would exceed the scope of this chapter. Nevertheless, some of its laws are important for this work. They are mentioned without further explanation in the course of this section, but a deeper insight into their foundations and additional sources are given in Appendix A.1.

In the following, first of all the models of the pipes are presented. Afterwards, the various elements the substations consists of are modeled. These are the valves, the pumps (pressure dictation pump and flow pump) and the heat exchanger. These are used in a final step to model the producer and the consumer substation.

The pipes constitute the connection between the other subsystems. They consist mostly of steel with a plastic insulation and therefore have a rather low elasticity. Plastic pipes are also used, but to a lesser extent due to their limitations in maximum temperature and pressure [Nus20, pp. 60ff.]. Under consideration of the falling network temperatures in DHNs mentioned in Section 1.2, however, their deployment becomes more feasible in future.

For the modeling of the pipes, two fundamental relationships from Appendix A.1 can directly be used. The first one is the Euler equation. This differential equation gives the dynamics of a fluid in a fluid filament, i.e. in this work a pipe.

$$\frac{\partial q_l}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} \tag{3.1}$$

This equation uses the volume flow rate q_l, the density of the fluid ρ, the pressure p and the spatial coordinate s.

The other relationship describes the friction in a pipe, which is dependent on the volume flow and can be given with a simple equation. The parameter λ comprises several parameters that are lumped together as explained in Appendix A.1.4.

$$\Delta p = \operatorname{sgn}(q)\lambda q^2. \tag{3.2}$$

By using the relationship for the pipe friction and Eq. (3.1) at the two ends of the pipe, it follows

$$\begin{aligned} \Delta p &= p_{\text{in}} - p_{\text{out}} \\ &= \operatorname{sgn}(q_l)\lambda_l q_l^2 + \rho\frac{\partial q_l}{\partial t} \\ &= \lambda_l(q_l) + L_l \dot{q}_l \end{aligned} \tag{3.3a}$$

According to the theory of PHSs introduced in Section 2.4.3, it is possible to write Eq. (3.3a) as a PHS with the state

$$x_l := L q_l \tag{3.3b}$$

and the storage function

$$H(x_l) = \frac{x_l^2}{2L}. \tag{3.3c}$$

Since the input and output pressure p_{in} and p_{out} cannot be controlled directly, they can be classified as uncontrolled interaction (coupling) inputs and the whole PHS arises as

$$\begin{aligned} \dot{\boldsymbol{x}}_l &= -\lambda_l(q_l) + \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} p_{\text{in}} \\ p_{\text{out}} \end{pmatrix} \\ &= -R(x_l) + \boldsymbol{K}\boldsymbol{z} \end{aligned} \tag{3.3d}$$

$$\boldsymbol{d} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \nabla H(x_l) = \begin{pmatrix} q_l \\ -q_l \end{pmatrix}. \tag{3.3e}$$

In terms of an equivalent electric circuit diagram, the pipe can be represented as in Figure 3.1. The model is equivalent to the pipe model used in [Str+21].

Remark 19: The elasticities of the pipes are not considered in this model. It could be interesting to include them in future works for two reasons: First of all, the pipe system may not be good enough ventilated and if air is present in the network, affects the (in)compressibility. Additionally and as mentioned earlier in this section, future DHNs could use more plastic pipes. These have an expansion coefficient 20 times as high as steel pipes [Nus20, p. 133] and therefore could become relevant in upcoming systems. Possibly, the elasticities could be lumped into the

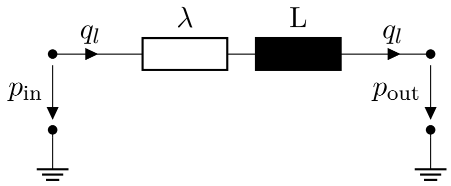


Figure 3.1: Equivalent circuit of a hydraulic pipe model.

elasticities of the substations that are discussed later on in this chapter.

3.1.6 Single Elements of the Producer and Consumer Substation

With the pipes, the first subsystem of a DHN has already been modeled. Additionally, the equations for the producer and the consumer substation have to be formulated. These systems consist of several elements which are considered in the following. When this is done, the subsystems can be composed out of the elements and then be modeled.

Valves

Valves control the flow of fluids by opening and closing completely or in parts. Their characteristic curve describes the connection between pressure drop over the valve and the flow through it and is often nonlinear. Consequently, they can be seen as nonlinear, controllable hydraulic resistors. Their characteristic curve is a quadratic relationship between pressure and flow [Nus20, ch. 7.2] and therefore a form can be found that is very similar to Eq. (3.2). The only additional term is the opening degree respectively valve lift ratio $r_{\text{lift}} \in [0, 1]$ and a mathematical description follows as

$$\Delta p = \text{sgn}(q)\lambda_v(r_{\text{lift}})q^2. \tag{3.4}$$

Remark 20: It shall be noted that valves are characterised on the basis of the k_v-value in industrial settings. This quantity is inverse to the friction coefficient and follows the correlation [Nus20, ch. 8.4.3]

$$k_v = \frac{q_{\text{max}}}{\sqrt{\Delta p}}$$

which is, disregarding the sign function, eventually equivalent to Eq. (3.4).

Remark 21: Several valve designs exist which have different equations of motion for the opening disk [Nus20, ch. 8.4.3]. Due to this fact, the value r_{lift} is used that represents the output of this motion equations. Since $\lambda_v(r_{\text{lift}})$ is always positive, generality is secured.

Pumps

Pumps create the flow in a pipe system and a pressure increase between their input and output. In DHNs, commonly electrically driven centrifugal pumps, also called circulating pumps, are used [Nus20, p. 45]. They accelerate the fluid in a rotating pump wheel and press it against the pump walls. By this means, the fluid begins to flow from the input in the middle of the pump wheel to

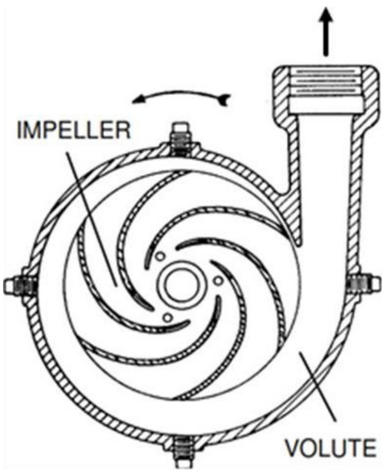


Figure 3.2: Schematic diagram of a centrifugal pump. Taken from [Has15].

the output at the side. A higher pressure appears at the output than at the input. The operating principle of centrifugal pumps is depicted in Figure 3.2.

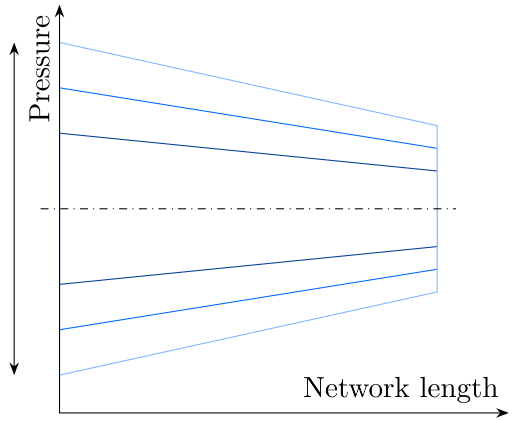
Two things follow directly from the design of the centrifugal pumps. Firstly, they can only transport fluids from the input in the middle to the output at the side, i.e. in one direction.

The second point needs to be elaborated a bit more: The fluid enters the pump in the middle of the pump wheel and is subsequently accelerated. Therefore a point of low pressure is created at the input where the fluid is "sucked away". This can lead to a problem called *cavitation* which occurs at high velocities and low pressure levels. Small gas bubbles are created in the fluid which eventually burst and damage the surrounding. This can drastically reduce the lifespan of pumps and therefore should be avoided [SA19, p. 263]. As cavitation appears at low pressures and high velocities and changing the velocity would affect the heat transport, the velocity cannot be changed. Instead, the pressure has to be held over a certain level for a secure pump operation. Since the point of lowest pressure in the pump lies at the input of the pump, it is necessary to take care of the input pressure.

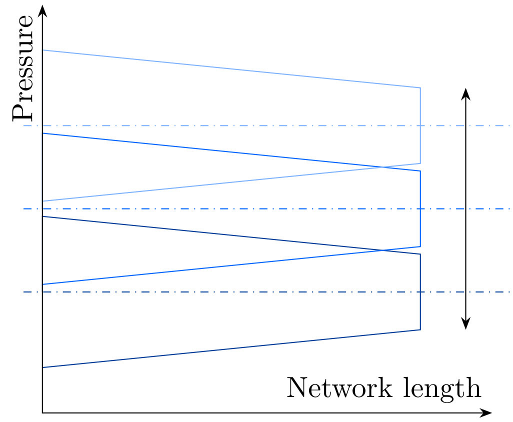
The problem of cavitation is in parts already the motivation for the pressure control (see [Nus20, p. 53] or Section 3.1.2). Since it may appear already at ambience pressure at certain velocities, an active control is needed to avoid damaging the pumps. A simple connection to the outside instead is not sufficient. Additionally, too high pressures can make the pipes burst or at least damage them. In consequence the pressure has to be kept in a rage that that does not allow it to become too high or too low. As mentioned in Section 3.1.2, this can be done either with a pump or a gas vessel.

> **Remark 22:** If the whole pipe network has no connection to the outside and therefore the ambient pressure, the absolute pressure level is not defined. In metaphorical sense, a pressure control can thus be seen as a "grounding" to the ambient pressure.

In consequence, two types of pumps are present in a DHN. The first type are the ones which create a volume flow, also called "flow pumps". They are built in the pipe loop, have no connection to the outside and are characterized by high flow rates. With a rising pressure difference, they create higher flow rates and bigger pressure losses at the pipes and remaining components. The other pumps are the pumps for pressure control which are connected to the surrounding and typically carry little fluid. Their purpose is to lift the whole pressure level in the network. The influence both types have on the pressure within the network is depicted in Figure 3.3.



(a) A pump inside a closed loop can create varying pressure differences that result in corresponding flow rates.



(b) A pump for pressure control cannot influence the pressure differences. Instead, it elevates or lowers the overall pressure in relationship to the ambient pressure.

Effects of pumps for fluid transport and for pressure control.

Figure 3.3: Effects of Pumps for Fluid Transport and for Pressure Control

Remark 23: In Section 3.1.2, another special type of pumps has been mentioned, the booster pumps. While most of the pumps in a DHN are placed at a household or a producer, booster pumps are built in the middle of the network. The reason for this installation is that in longer networks, the pressure of a small number of pumps is not sufficient to assure a flow in remote parts of the network. Otherwise, the pressure at the pumps had to be so high that it would exceed the admissible maximum pressure of the pipes. Therefore, pumps are installed in the middle of the network in order to create the desired pressures and flows [Nus20, p. 54]. A schematic can be found in Figure 3.4. Booster pumps could likewise be used in 4GDHN and 5GDHN to support the desired pressure and flow levels.

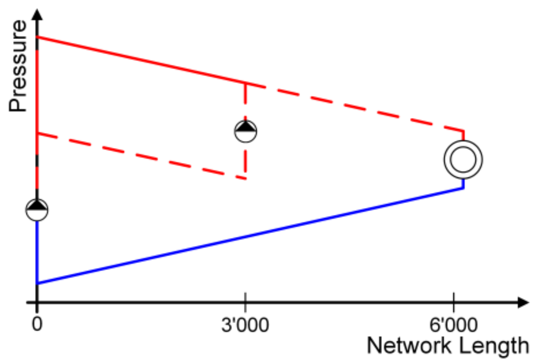


Figure 3.4: Example pressure curve with and without a booster pump. Taken from [Nus20].

Booster pumps are not explicitly treated in this work. But following [Mac+22], a PI control comparable to Proposition 11 is able to set the desired volume flow in a passive manner. The closed-loop system consisting of a booster pump and a PI control can then be included in the DHN while preserving the asymptotic stability of the desired equilibrium.

After regarding the different properties of the pumps in DHNs, their mathematical form is now sought. The pump model used is essentially equal to the one in [Vie, p. 58], even though this one is a bit more in-depth researched.

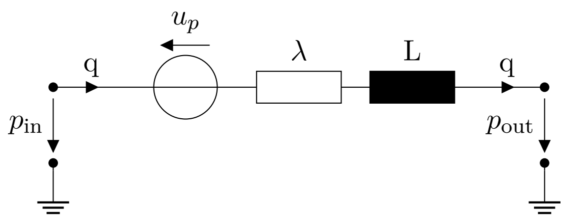


Figure 3.5: Equivalent circuit diagram of a centrifugal pump.

Centrifugal pumps are difficult systems to model because of the various geometric influences and the complexity of the equations describing the internal flows. In fact, an exact analytical calculation of the pump dynamics is regarded to be impossible [Sig21, p. 253]. Nonetheless, an approximate description is conceivable that includes important system features and retains a mathematical form of moderate difficulty.

A centrifugal pump as in Figure 3.2 can be regarded as a constant pressure source with a flow-dependent characteristic throttling and a dynamical behaviour resulting from the pump wheel [Sig21, pp. 253 - 257]. All components summed up yield the characteristic curve of the pump.

The loss curve in centrifugal pumps can be separated into two parts, the hydraulic losses and the shock losses [Sig21, pp. 253 - 257]. The hydraulic losses include influences such as friction or vortex losses and exist all the time. The shock losses instead appear only in cases with forced, discontinuous changes in velocity [Sig21, pp. 242] and shall be neglected according to assumption A1 and Section 3.1.3.

Hydraulic losses cannot be calculated exactly and are therefore determined by collecting data from a model and a curve fitting. The results show that usually the pressure loss Δp correlates to the flow rate q with $\Delta p \sim q^n$ and an exponent $n = 1.8 \ldots 2$ [Sig21, pp. 253 - 257]. For the reason of a margin of safety and easier calculation it is common to calculate with the quadratic law

$$\Delta p_{\text{loss}} = \lambda_p q^2. \tag{3.5}$$

In this case, the hydraulic losses are analog to the hydraulic losses of the pipes. Friction losses of the pipes leading into and out of the pump can therefore be incorporated into the parameter λ_p.

The dynamical parts of the pump can likewise easily be modeled analog to the dynamical behaviour of the pipes. The water flowing through the pump has naturally the equivalent correlation and the pump wheel with its rotational inertia follows the same principle. The dynamical parts of the pump accordingly follow as

$$\Delta p_{\text{dynamic}} = L_p \dot{q}$$

and the whole pump dynamic can be written as

$$\begin{aligned}\Delta p &= p_{\text{out}} - p_{\text{in}} \\ &= u_p + \lambda_p(q) + L_p \dot{q}.\end{aligned}$$

This corresponds to an electrical equivalent circuit diagram as shown in Figure 3.5. A PHS formulation can be found with the state

$$x = Lq,$$

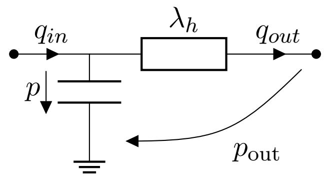


Figure 3.6: Equivalent circuit diagram of a heat exchanger.

the storage function

$$H(x) = \frac{x^2}{2L}$$

and the equations

$$\begin{aligned} \dot{x} &= -\lambda_p(q) - u_p + \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} p_{\text{in}} \\ p_{\text{out}} \end{pmatrix} \\ &= -R(x) - u_p + \boldsymbol{K}\boldsymbol{z} \end{aligned} \tag{3.6}$$

$$y = -\nabla H(x) = -q. \tag{3.7}$$

This modeling is valid both for the pumps for fluid transport and the ones for pressure control.

Heat exchangers

The last DHN subsystems considered are heat exchangers. Usually the hot water from the DHN does not heat the house directly but over intermediary installations [Nus20, pp. 86ff.]. From the perspective of the house installation, the DHN is seen as the primary circuit which flows in the heat exchanger. There, the heat is transmitted into the secondary local circuit that distributes the heat in the house. The advantage with reference to the modeling is that this is a clean and unified interface and everything behind the heat exchanger is not required to be modeled.

According to Section 3.1.3, the heat exchangers are the only parts of the network in which the elasticity has to be regarded. The elasticity can be measured by quantifying the difference of the water volume in normal operation pressure and e.g. 1 bar and calculating the ratio $C = \frac{\Delta V}{\Delta p}$ [BT17].

Besides, a heat exchanger has a hydraulic resistance equal to a normal pipe so it can be shown as an equivalent circuit as in Figure 3.6. This can be formulated as [Vie, p. 61]

$$\begin{aligned} C\dot{p} &= q_{\text{in}} - q_{\text{out}} \\ p &= -\lambda(q_{\text{out}}) + p_{\text{out}} \end{aligned}$$

Remark 24: Heat exchangers carry water and therefore have also a small inertia. With regard to the small amount of water flowing through a heat exchanger this can be neglected. Alternatively, as a heat exchanger is always built in in connection with a pump, this inertia can be lumped together with the inertia of the pump.

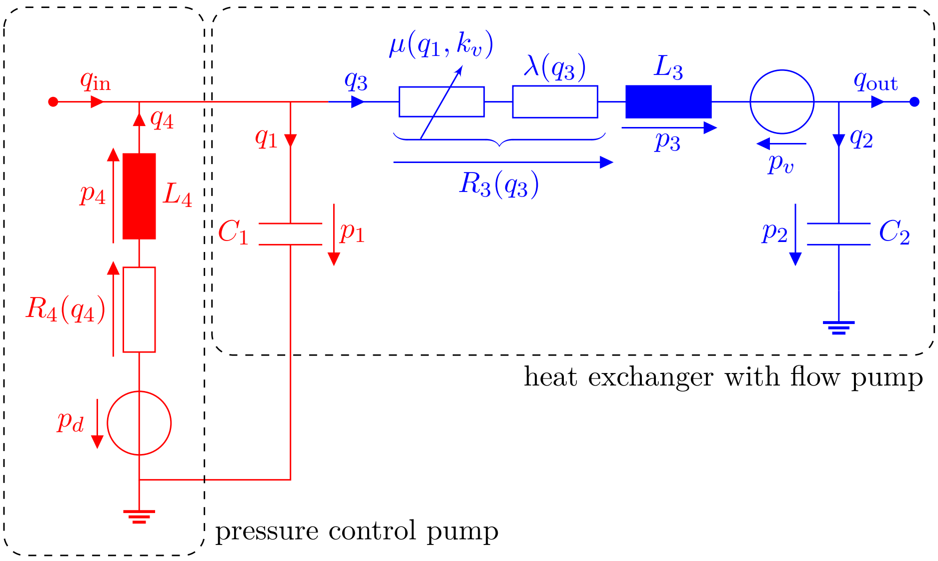


Figure 3.7: The producer substation. The boxes correspond to the physical systems, the colors to how the model is split up for control.

3.1.7 Producer Substations

The last section was designated to the modeling of the single components of a DHN. In the following, these results are used to model the elements of a DHN. The pipes have already been modeled in Section 3.1.5, so the only things that remain are the producer and consumer substations.

A complete producer substation consists of a pump, a valve, a heat exchanger and a pressure dictation pump. If all components (such as inertias or elasticities) that lie next to each other in the EEC are lumped together, the model of the producer substation can be shown as in Figure 3.7. This modeling bases on the models in [Str+21], but it is extended by the pressure dictation units.

The model equations can be given as described in the following: In line with Eq. (3.2), the relationships

$$R_i(q_i) = \operatorname{sign}(q_i)\lambda_i q_i^2, \qquad i = 3, 4 \tag{3.8}$$

apply.

From Kirchhoffs Current and Volatage Laws, the following relationships result:

$$\begin{aligned}
q_{\text{in}} + q_4 &= q_3 + q_1 && \Leftrightarrow q_1 = -q_3 + q_4 + q_{\text{in}} \\
q_3 &= q_2 + q_{\text{out}} && \Leftrightarrow q_2 = q_3 - q_{\text{out}} \\
p_1 + p_v &= R_3(q_3) + p_3 + p_2 && \Leftrightarrow p_3 = p_1 - p_2 - R_3(q_3) + p_v \\
R_4(q_4) + p_4 + p_1 &= p_p && \Leftrightarrow p_4 = -p_1 - R_4(q_4) + p_p
\end{aligned}$$

With

$$\begin{aligned}
\boldsymbol{x} &= \begin{pmatrix} C_1 p_1 & C_2 p_2 & L_3 q_3 & L_4 q_4 \end{pmatrix}^\top \\
&= \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^\top \\
H(\boldsymbol{x}) &= \frac{x_1^2}{2C_1} + \frac{x_2^2}{2C_2} + \frac{x_3^2}{2L_3} + \frac{x_4^2}{2L_4} \\
\nabla H^\top(\boldsymbol{x}) &= \begin{pmatrix} \frac{x_1}{C_1} & \frac{x_2}{C_2} & \frac{x_3}{L_3} & \frac{x_4}{L_4} \end{pmatrix}^\top = \begin{pmatrix} p_1 & p_2 & q_3 & q_4 \end{pmatrix}^\top
\end{aligned} \tag{3.9}$$

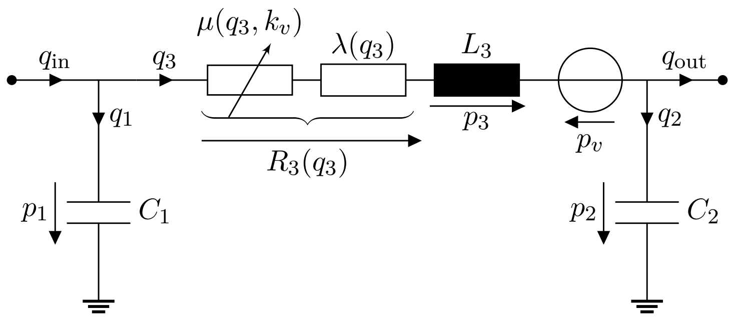


Figure 3.8: Equivalent circuit of a customer substation.

the following ISO-PHS system follows

$$\dot{\boldsymbol{x}} = \begin{pmatrix} q_1 \\ q_2 \\ p_3 \\ p_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}}_{=\boldsymbol{J}} \underbrace{\begin{pmatrix} p_1 \\ p_2 \\ q_3 \\ q_4 \end{pmatrix}}_{=\nabla H(\boldsymbol{x})} - \underbrace{\begin{pmatrix} 0 \\ 0 \\ R_3(q_3) \\ R_4(q_4) \end{pmatrix}}_{=\mathcal{R}(\boldsymbol{x})} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{=\boldsymbol{G}} \underbrace{\begin{pmatrix} p_v \\ p_p \end{pmatrix}}_{=\boldsymbol{u}} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}}_{=\boldsymbol{K}} \underbrace{\begin{pmatrix} q_{\text{in}} \\ q_{\text{out}} \end{pmatrix}}_{=\boldsymbol{d}} \tag{3.10a}$$

with the natural passive output

$$\boldsymbol{y} = \boldsymbol{G}^\top \nabla H(\boldsymbol{x}) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \nabla H(\boldsymbol{x}) = \begin{pmatrix} q_3 \\ q_4 \end{pmatrix} \tag{3.10b}$$

and the uncontrollable outputs

$$\boldsymbol{z} = \boldsymbol{K}^\top \nabla H(\boldsymbol{x}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \nabla H(\boldsymbol{x}) = \begin{pmatrix} p_1 \\ -p_2 \end{pmatrix}. \tag{3.10c}$$

3.1.8 Customer Substations

The consumer substations are modelled equally to the models in [Str+21], but the pressure expansion vessels are taken into account. A vessel can be seen as a capacitor respectively an elasticity and it lies in parallel to the elasticities which come from the heat exchanger. Therefore they can be lumped together. Figure 3.8 shows that thus the resulting model looks equal to the model from [Str+21] on which it is based on. It can also be seen that it corresponds to the producer substation when $R_4, L_4 \to \infty$ resp. $x_4 \to 0$.

For a PHS modeling, the state and the Hamiltonian can be given as

$$\begin{aligned} \boldsymbol{x} &= \begin{pmatrix} C_1 p_1 & C_2 p_2 & L_3 q_3 \end{pmatrix}^\top \\ H(\boldsymbol{x}) &= \frac{x_1^2}{2C_1} + \frac{x_2^2}{2C_2} + \frac{x_3^2}{2L_3} \\ \nabla H^\top(\boldsymbol{x}) &= \begin{pmatrix} \frac{x_1}{C_1} & \frac{x_2}{C_2} & \frac{x_3}{L_3} \end{pmatrix}^\top = \begin{pmatrix} p_1 & p_2 & q_3 \end{pmatrix}^\top. \end{aligned} \tag{3.11}$$

The ISO-PHS equations follow as

$$\dot{\boldsymbol{x}} = \begin{pmatrix} q_1 \\ q_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ q_3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ R_3(q_3) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} p_v + \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} q_{\text{in}} \\ q_{\text{out}} \end{pmatrix} \tag{3.12a}$$

with the natural passive output

$$y = \boldsymbol{g}^\top \nabla H(\boldsymbol{x}) = q_3 \tag{3.12b}$$

and the uncontrollable outputs

$$\boldsymbol{z} = \boldsymbol{K}^\top \nabla H(\boldsymbol{x}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \nabla H(\boldsymbol{x}) = \begin{pmatrix} p_1 \\ -p_2 \end{pmatrix} \tag{3.12c}$$

3.2 System Analysis

As stated in Section 1.4, the aim of this work is to provide a stable, extensible control for DHNs via passivity-based control. From the PHS structure follows passivity with respect to $\underline{x} = \underline{0}$. Of course the goal is not to achieve it towards this equilibrium, but to any possible one. Therefore the admissible equilibria and all passive outputs are analysed. Important properties like zero-state observability are investigated and finally it is discussed, how this affects the controller design, which controllers are possible and which ones are advantageous.

3.2.1 Admissible Equilibria

Producer Substation

For the desired equilibrium to be set, Eq. (2.36) has to be satisfied. This follows for the complete system (3.10a) as

$$\begin{aligned} \boldsymbol{x}^* &= \{\boldsymbol{x} \in \mathbb{R}^4 | \boldsymbol{g}^\perp \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}\} \\ &= \left\{\boldsymbol{x} \in \mathbb{R}^4 \middle| \boldsymbol{g}^\top \left(\boldsymbol{J} \nabla H(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})\right) = \boldsymbol{0}\right\} \\ &= \left\{\boldsymbol{x} \in \mathbb{R}^4 \middle| \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}^\top \left(\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ q_3 \\ q_4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ R_3(q_3) \\ R_4(q_4) \end{pmatrix} \right) = \boldsymbol{0} \right\} \\ &= \left\{\boldsymbol{x} \in \mathbb{R}^4 \middle| \begin{pmatrix} p_1 - p_2 - R_3(q_3) \\ -p_1 - R_4(q_4) \end{pmatrix} = \boldsymbol{0}\right\} \end{aligned} \tag{3.13}$$

This restricts the set off possible equilibria to a plane in 4 dimensions with two degrees of freedom. Accordingly, either two flow rates through the inductors, two pressures over the capacitors or one of each can be freely chosen. This relates to intuition and the fact that two actuators are present in the circuit.

Consumer Substation

In the case of the consumer substation, the results look similar. Eq. (2.36) yields here only the first line of Eq. (3.13):

$$\begin{aligned}\boldsymbol{x}^* &= \{\boldsymbol{x} \in \mathbb{R}^3 | \boldsymbol{g}^\perp \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}\} \\ &= \left\{\boldsymbol{x} \in \mathbb{R}^3 \middle| \boldsymbol{g}^\top \left(\boldsymbol{J}\nabla H(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})\right) = \boldsymbol{0}\right\} \\ &= \left\{\boldsymbol{x} \in \mathbb{R}^3 \middle| p_2 - p_1 - R_3(q_3) = 0\right\} \end{aligned} \tag{3.14}$$

3.2.2 Zero-state Detectability and Observability

Producer Substation

In order to determine the observability of the system (3.10a), the positive invariant set with $\boldsymbol{y} = \boldsymbol{u} = \boldsymbol{0}$ has to be determined (Section 2.2). Directly follows with Eq. (3.10b)

$$\boldsymbol{y} = \boldsymbol{G}^\top \nabla H(\boldsymbol{x}) = \begin{pmatrix} q_3 \\ q_4 \end{pmatrix} \overset{!}{=} \boldsymbol{0} \tag{3.15}$$

and these results together with the state-space equation Eq. (3.10a) result in

$$\boldsymbol{y} = \boldsymbol{u} = \boldsymbol{0} \Rightarrow \boldsymbol{x} = \boldsymbol{0} \tag{3.16}$$

This means that the producer system is zero-state observable.

Consumer Substation

At this point, differences between the producer and the consumer substation appear. Eq. (3.12b) yields indeed the same result

$$y = q_3 \overset{!}{=} 0,$$

but from this relationship only follows together with Eq. (3.12a)

$$\dot{\boldsymbol{s}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ R_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ p_1 - p_2 \end{pmatrix} \tag{3.17}$$

This means that p_1 and p_2 resp. x_1 and x_2 are constant and both have the same offset to zero. Accordingly, there is no zero-state observability.

> **Remark 25:** Albeit missing observability normally may point towards an error in the modeling process, this is not the case here: The purpose of the pump in the consumer substation is only to create pressure differences which propel the flow in the system. It was never designed to set absolute pressure levels because the pressure differences are independent from the absolute level. Consequently, the offset that follows from Eq. (3.17) reflects this design choice. The producer substation in contrast has two pumps, one for creating the flow and one for holding the pressure at a desired level, and is therefore zero-state observable.

> Another way to look at this is to remark that the two capacitors in Figure 3.8 have no influence on the current in a steady state since the whole connection to the network takes place with an voltage offset.

Fortunately, observability or detectability, though beneficial features, are not necessary for a successful control. As results later on show, the observability of the producer substations causes also the consumer substations to become observable in the interconnected grid.

3.2.3 Parameterization of all Passive Outputs

According to [Jay+07], every passive output of a system can be controlled with a PI-controller. The outputs of the PHS formulation of the producer subsystem - taking the interaction ports into account - are according to Eqs. (3.10b) and (3.10c)

$$\boldsymbol{y}_{\text{natural passive}} = \begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} q_3 \\ q_4 \\ p_1 \\ -p_2 \end{pmatrix},$$

so by implication of Lemma 2 all of the states are passive outputs of the system. But taking into account that the passivity inequality is formulated as $\dot{S} \leq \boldsymbol{u}^\top \boldsymbol{y}$ (see Definition 10), not every output is passive with respect to every input. Instead, only the i-th output is passive to the i-th input which means that a passivity relation exists between $\boldsymbol{u}$ and $\boldsymbol{y}$ and between $\boldsymbol{d}$ and $\boldsymbol{z}$, but not e.g. between $\boldsymbol{u}$ and $\boldsymbol{z}$. Therefore, a simple PI-control is not directly possible neither for the grid-feeding, nor for the grid-forming mode as both are in need of the control of p_1.

Indirectly however, many more passive input-output-connections can exist for PHSs and the parameterization of all passive outputs is given by Proposition 2. In this section, the results are applied on the given systems and the practical feasibility of the method is assessed.

The first step to parameterize all passive outputs is to decompose $\boldsymbol{R}(\boldsymbol{x})$ with $\mathcal{R}(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{x})\nabla H(\boldsymbol{x})$ into

$$\boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{\phi}^\top(\boldsymbol{x})\boldsymbol{\phi}(\boldsymbol{x}). \tag{2.29 revisited}$$

From Eq. (2.36) follows

$$\boldsymbol{R}(\boldsymbol{x}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \operatorname{sign}(q_3)\lambda_3 q_3 & 0 \\ 0 & 0 & 0 & \operatorname{sign}(q_3)\lambda_3 q_3 \end{pmatrix}.$$

Since $\boldsymbol{\phi}(\boldsymbol{x}) \in \mathbb{R}^{q \times n}$ with q to be freely chosen, the set of possible $\boldsymbol{\phi}(\boldsymbol{x})$ is infinite. But as $\boldsymbol{R}(\boldsymbol{x})$ is a diagonal matrix, all of the elements of this set can be given by a matrix with the column vectors $\boldsymbol{v}^{(1)}$ and $\boldsymbol{v}^{(2)}$

$$\boldsymbol{\phi}(\boldsymbol{x}) = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{v}^{(1)} & \boldsymbol{v}^{(2)} \end{pmatrix}$$

where

$$\sum_{i=1}^{q} \boldsymbol{v}_i^{(1)} = \sqrt{\operatorname{sign}(q_3)\lambda_3 q_3}$$

$$\sum_{i=1}^{q} \boldsymbol{v}_i^{(2)} = \sqrt{\operatorname{sign}(q_4)\lambda_4 q_4}$$
$$(\boldsymbol{v}^{(1)})^\top \boldsymbol{v}^{(2)} = 0.$$

Proof This follows from the sum representation of a matrix multiplication $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{ik}$: $\boldsymbol{R}(\boldsymbol{x})$ is zero on the first two diagonal elements and using Eq. (2.29) and $\boldsymbol{\phi}(\boldsymbol{x}) = (\varphi_{ij}), \varphi_{ij} \in \mathbb{R}$

$$\boldsymbol{R}_{i,i}(\boldsymbol{x}) = 0 = \sum_{k=1}^{q} \varphi_{ik}^2, \qquad i = 1, 2.$$

Therefore all entries of $\boldsymbol{\phi}(\boldsymbol{x})$ in the first and second column have to be zero. The residual statements can be shown in a similar way. ∎

Remark 26: Notice that $\operatorname{sign}(q_3)\lambda_3 q_3 \geq 0$ and thus the square root exists for all q_3. The same applies for $\operatorname{sign}(q_4)\lambda_4 q_4$.

With the freely selectable parameter $\boldsymbol{\omega}(\boldsymbol{x}) = \begin{pmatrix}\boldsymbol{\omega}^{(1)} & \boldsymbol{\omega}^{(2)}\end{pmatrix} \in \mathbb{R}^{q \times m} = \mathbb{R}^{q \times 2}$ the first part of the passive output follows to

$$\boldsymbol{h}(\boldsymbol{x}) = \left[\boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{\phi}^\top(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x})\right]^\top \nabla \boldsymbol{H}(\boldsymbol{x}) \tag{2.27 revisited}$$

From this point on, three options exist: If $\boldsymbol{\omega}(\boldsymbol{x}) = \boldsymbol{0}$ is selected, the output corresponds to the already mentioned natural passive output and with this selection, no additional passive outputs can be found.

In the second case, the aim is to have the pressures p_1 and p_2 in the output without the flows q_3 and q_4. This means that $\boldsymbol{\omega}(\boldsymbol{x})$ needs to contain elements with a factor $\frac{1}{\sqrt{\operatorname{sign}(q_3)\lambda_3 q_3}}$ so that the product $\boldsymbol{\phi}^\top(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x})$ becomes one and a freely selectable function can be included in $\boldsymbol{\omega}(\boldsymbol{x})$. But Proposition 2 states the output to be of a form

$$\boldsymbol{y}_{\mathrm{wD}} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{j}(\boldsymbol{x})\boldsymbol{u} \tag{3.18}$$

with

$$\boldsymbol{j}(\boldsymbol{x}) = \boldsymbol{\omega}^\top(\boldsymbol{x})\boldsymbol{\omega}(\boldsymbol{x}) + \boldsymbol{D}(\boldsymbol{x}) \tag{2.28 revisited}$$

so this can only eliminate q_3 and q_4 from $\boldsymbol{h}(\boldsymbol{x})$, not from $\boldsymbol{y}_{\mathrm{wD}}$.

The third option is to use any other $\boldsymbol{\omega}(\boldsymbol{x})$, but in this case q_3 and q_4 will always be present in the output due to $\boldsymbol{\phi}(\boldsymbol{x})$. Therefore, the connection of one of the accessible inputs to the outputs p_1 and p_2 can never be passive and the control via a PI-controller is not possible with a passivity-based control approach. In contrast, as mentioned in [Mac+22], the flows can easily be controlled via PI-control.

3.2.4 Feasibility of IDA-PBC

IDA-PBC is one of the most versatile control methodologies for nonlinear systems [OGC04], yet it is not every time applicable. In the case of this system, it can be shown that despite the many degrees of freedom that this method allows, no IDA-PBC control law can be found to control the flow rate q_3 and the pressure p_1, i.e. a grid-feeding controller.

In order to prove this, several observations have to be made: The first one is that the system has two HRD states, namely x_1 and x_2. The dynamics of these states cannot be changed directly, resp. the state equations will stay the same regardless of the control law. If it is intended to control HRD states, these can only be affected indirectly through the equations of RD1 states.

Apart from the physical idea, this can also be seen directly in the standard PHS state function Eq. (2.22): Depending on the input $\boldsymbol{u}$, the RD1 state equations can be written with any desired correlation, assuming a complete compensation of all dynamics. The HRD state equations in contrast stay the same[1].

So after applying a IDA-PBC control with arbitrary dynamics $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ and full compensation of all RD1 dynamics, the state equation of the resulting system can become (ref. the original Eq. (3.10a))

$$\dot{\boldsymbol{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} q_4 - q_3 \\ q_3 \\ f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \end{pmatrix} \tag{3.19}$$

The second line of this matrix equation shows that if q_3 takes a stationary value different from zero, the pressure $p_2 = \frac{x_2}{C_2}$ over C_2 can never be stationary. IDA-PBC is therefore not applicable for this system. Since this error-causing second line is identically present in the state equation of the consumer substations, the result is the same for both DHN subsystems.

While this result could be interpreted as the proof of the impossibility of a control law for the producer, this is not the case. Instead, it shows a weakness of the IDA-PBC method which cannot easily be overcome.

IDA-PBC, on a basic level, uses the complete dynamics of an islanded model without the interconnection ports to set a certain equilibrium. “The complete dynamics” means it uses the actual physics if possible and compensates them if necessary, afterwards posing different dynamics. The resulting still islanded system is targeted to have a constant state.

This is already the main problem: On the one hand, a connection to other systems is not supposed to exist and therefore no external dynamics are taken into account. On the other hand, the complete physical dynamics of the islanded system are used which may not represent the whole system in interconnection. This leads to the problem that the desired setpoint is not available in this mathematical framework.

> **Remark 27:** This observation becomes obvious with respect to the circuit diagram Figure 3.7. The physics of this islanded model do not allow that there is a constant voltage over the capacitor while at the same time the currents q_3 and q_4 are different from zero.

The results in turn do *not* show two things: They do not show that no control with IDA-PBC can be designed for this system. If the goal is for example to develop a grid-forming controller that aims at controlling the pressures p_1 and p_2, IDA-PBC is one possible way. Also, since the RD1 state equations can be freely chosen, there is no direct limitation for this states.

Furthermore, following results of this work will show that the system can be controlled in flow and pressure as discussed here. These results however will show that the problem is feasible if the system is considered on a signal level rather than from a very physical, port-based view.

[1] Of course the resulting PHS of an IDA-PBC-controlled system can be reformulated to get the desired form Eq. (2.44), but this does not change the underlying dynamics of the HRD state equations. Instead, only components are equally added as well as subtracted in order to change the mathematical form.

Remark 28: Although in the following a possible control will be shown to exist, this result shows a minor limitation of it: The admissible states of all subsystems cannot be chosen independently. Once again regarding the second line of the original state equation of the producer subsystem Eq. (3.10a)[2] shows

$$q_3 = q_{\text{out}}$$

for x_2 to be at a constant state. This means the flow in the adjacent pipe $q_p = q_{\text{out}} = q_3$ has to be equal to the flow in the subsystem and relates to assumption A6 that states that the desired state of the network has to be chosen in a way that it exits in the connected grid. Of course from a practical perspective one could argue that nobody would have expected anything else.

3.2.5 Subdivision of the System

Because the grid-feeding control of the producer subsystem as a whole leads to the problems already described above, this section evaluates the possibility of splitting the system from one mutliple-input-multiple-output (MIMO) system into two single-input-single-output (SISO) systems. A natural subdivision is according to the physical systems, i.e. the pressure control pump on the one side and the heat exchanger and flow pump on the other side. Note that the latter subsystem is equal to the consumer substation. Unfortunately, using this option leads to problems:

At every connection of multiple systems, the compatibility conditions from Section 2.4.2 have to be fulfilled. This condition does not only hold as long as the system is regarded at a physical level, but also when the signal level is observed. This is due to the fact that a stability analysis of the controlled grid needs compatible port variables in order to apply the passivity theorem Proposition 1. The junction between the pressure control and the heat exchanger is a flow junction, so one neighbouring system has to define the effort, in this case the pressure, and the other systems have to define the flows at the conjunction. The three neighbouring systems are the pipe connecting the substation to the network, the pressure control and the combination of heat exchanger and flow pump.

According to Eq. (3.3e), the pipe takes the pressures at both ends as input and the flow as the output. The heat exchanger and flow pump, being equivalent to the consumer substation, outputs the pressure p_1 over the capacitor as reported by Eq. (3.12b). The output of the pressure control pump is the flow rate q_4 through the inductor L_4. Therefore, as it can be expected from a physical system, one system defines the effort at the junction and the other ones the flows and hence the compatibility conditions are met for a standard PHS modeling.

The problem that arises here is however directly connected to this fact: Since the pressure control does not output the pressure in a signal level representation, it is not possible to find a control law for it that controls the pressure. A naive alternative would be to reformulate the output equation of the pressure control. But this leads to multiple problems since on the one hand a derivative term would be needed to calculate the voltage over the inductor. On the other hand, this would contradict the compatibility conditions since the effort at the junction is already defined by the consumer substation-like system.

So the pressure control pump apparently can manipulate the pressure at the junction, but as the computational causality causes the outputs to be as described, this model of it cannot control it directly. Otherwise the laws of causality and compatibility are violated, or even worse, the system of the heat exchanger and flow pump also has to be reformulated and problems with computational causality arise also there.

[2]This equation, of course, will hold independently from the control law used. It is a physical fact and cannot be changed. A control law can influence it from outside, but it has to respect the underlying physics.

A more advantageous way to model the producer substation is to divide it into two parts as depicted in Figure 3.7. The parts can be given as follows:

Definition 21: The producer substation can be split up in two parts called system 1 and system 2. System 1 consists of the pressure control pump and the capacitor C_1 of the heat exchanger and is colored red in Figure 3.7. System 2 consists of the heat exchanger and the flow pump without the capacitor C_1 which is allocated to the first system. It is the blue part of the producer substation.

So the difference to the previously discussed modeling is small and reduces to only reattributing C_1 to the other subsystem. As will be shown below, the differrence is however significant: In this case, the natural passive output of system 1 stays the flow q_4 but the output of system 2 becomes q_3. System 1 can be given as

$$\dot{\boldsymbol{x}} = \begin{pmatrix} C_1\dot{p}_1 \\ L_4\dot{q}_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{\boldsymbol{J}} \nabla H(\boldsymbol{x}) - \underbrace{\begin{pmatrix} 0 \\ R_4(q_4) \end{pmatrix}}_{\boldsymbol{R}(\boldsymbol{x})} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\boldsymbol{G}} \underbrace{p_D}_{u} + \underbrace{\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}}_{\boldsymbol{K}} \underbrace{\begin{pmatrix} q_3 \\ q_{\text{in}} \end{pmatrix}}_{\boldsymbol{z}} \tag{3.20a}$$

$$\boldsymbol{y} = \boldsymbol{G}^\top \nabla H = q_4 \tag{3.20b}$$

with the state $\boldsymbol{x}$ and storage function $H(\boldsymbol{x})$ being

$$\boldsymbol{x}^\top = \begin{pmatrix} x_1 & x_4 \end{pmatrix}^\top = \begin{pmatrix} C_1 p_1 & L_4 q_4 \end{pmatrix}^\top$$
$$H(\boldsymbol{x}) = \frac{x_1^2}{2C_1} + \frac{x_4^2}{2L_4}$$

and system 2 given as

$$\dot{\boldsymbol{x}} = \begin{pmatrix} C_2\dot{p}_2 \\ L_3\dot{q}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{\boldsymbol{J}} \underbrace{\begin{pmatrix} p_2 \\ p_3 \end{pmatrix}}_{\nabla H} - \underbrace{\begin{pmatrix} 0 \\ R_3(q_3) \end{pmatrix}}_{\boldsymbol{R}(\boldsymbol{x})} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\boldsymbol{G}} \underbrace{p_v}_{u} + \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{K}} \underbrace{\begin{pmatrix} q_{\text{out}} \\ p_1 \end{pmatrix}}_{\boldsymbol{z}} \tag{3.21a}$$

$$\boldsymbol{y} = \boldsymbol{G}^\top \nabla H = q_3 \tag{3.21b}$$

with the state $\boldsymbol{x}$ and storage function $H(\boldsymbol{x})$

$$\boldsymbol{x}^\top = \begin{pmatrix} x_2 & x_2 \end{pmatrix}^\top = \begin{pmatrix} C_2 p_2 & L_3 q_3 \end{pmatrix}^\top \tag{3.21c}$$

$$H(\boldsymbol{x}) = \frac{x_2^2}{2C_2} + \frac{x_3^2}{2L_3}. \tag{3.21d}$$

Remark 29: This does not directly fulfill the compatibility condition since with these models the equation of flows is over-determined (every system has a flow as output) and the pressure is undetermined (no system has the pressure as output). But now system 1 contains the state x_1 with the pressure at the junction and system 2 outputs a flow. Therefore it is possible to find another output of system 1 that gives the pressure at the node and at the same time the compatibility is secured and no differentiation is necessary.

Remark 30: From a less theoretic point of view, the result can also be explained differently: The capacitor at the junction determines the pressure at the junction. Therefore, as the pressure control pump is made for controlling this pressure, it must have a direct influence on it. This is not given if the two systems are controlled seperately and the control laws are calculated for the isolated subsystems so the capacitor has to be included into the system of the pressure control pump. Since this is only a reattribution, the continuity laws are always fulfilled on a physical level. However, now they are also accomplished for the signal level.

Remark 31: The two systems can now both be regarded as a RLC element with a nonlinear resistance. Their only differences are that in system 1 their elements are arranged in a closed loop while in system 2 they are in series. Additionally, the uncontrollable in- and outputs can possibly enter and leave system 1 completely through the wires without influencing the dynamics, but they always have an influence on R_3 and L_3 in system 2.

The volume flow of system 2 can be controlled with a PI control as it is the natural passive output. The pressure of the capacitors in both system 1 and system 2 in turn can possibly be controlled with IDA-PBC. This modeling thus solves the problems of the non-split model discussed in the previous sections. At the same time, it simplifies the control design by turning the MIMO model into two SISO systems. The remaining task is to find a suitable control law that ensures the passivity of both systems and controls them to the desired setpoints. This will be addressed in the next chapter.

Chapter 4

Controller Synthesis

A DHN as defined per Definitions 20 and 21 consists of producer and consumer substations and pipes that connect them. The pipes are uncontrollable but the substations can be controlled by the pressure differences that are caused by the built-in pumps. The control laws must create respectively maintain passivity for every subsystem and fulfill the objectives O1 - O4.

The following chapter provides an overview of the control structure in Section 4.1. The designs of the controllers are proposed in Section 4.2. A clear distinction is made to the stability analysis which is given in Section 4.3. The chapter closes with Section 4.4 where the developed controllers are analysed and compared to designs for electric microgrids.

4.1 Control Overview

Before the beginning of the control design, an overview of the different controllers shall be given in Section 4.1.1. Under the knowledge of the models, it is possible to refine the control concept proposed in Section 1.4. On a high level, it can be discussed how the systems will behave and what this means for the control objectives in Section 4.1.2. When these are defined definitely, it can be proposed with which methods the controllers should be developed.

4.1.1 Overview over the Controller

According to objectives O3 and O4, two types of controllers are to be developed, a grid-feeding and a grid-forming type. This approach can be justified retrospectively by the models: Both of them have a built-in pump that pumps in the direction of the volume flow. The flow determines the heat flow and therefore controllers for the volume flow are needed. These are the grid-feeding controller for the producer and the controller for the customer substation.

However, only the producer substation has an additional pump that can adjust the pressure directly. It is hence the only zero-state observable system in this network. The customer substations in contrast are equipped with a gas vessel that cannot influence the pressure actively. Two things follow from this: Firstly, the pressure at the producer substations has to be controlled. This ensures a safe operating of the flow pumps at the producer. But as the customer substations cannot ensure safe pressures directly, the grid-forming controller is necessary. This one sets the pressure at the output of the producer substations to a level that guarantees safe pressures at the inputs of the customer substations.

It makes intuitively sense to have at least one grid-forming controller in the network. However, with a look at the network it becomes clear that this condition alone is not sufficient, at least for a theoretical resp. mathematical treatment. In the following, it is presented why this is the case and how it can be determined which controller mode has to be used in which producer substation. How things look in practice, on the other hand, is discussed in Remark 35 and Appendix C.3.

4.1.2 Assignment of the Operating Modes

The following consideration is based on the assumption that suitable volume flow controllers and pressure controllers exist. In addition, it is assumed that there exists a target state consisting of known pressures and flow rates, which forms an equilibrium of the system. The system is operated in a state of equilibrium, which however (as will be shown) does not necessarily have to be the target state. This consideration is be made at this point in order to make clear what is possible with the controlled system at all. It is deliberately kept on a conceptual and verbal level, because this means that the controllers to be developed do not have to be known yet. However, this does not change the validity of the statements. The observations are binding independently of the actual realization of the pressure and volume flow controllers. In Section 4.3.3 is shown that they also appear in mathematics.

It turns out that

- each input and output of a consumer substation and
- each output of a producer substation in grid-feeding mode

must be continuously connected via pipes to a point whose pressure is fixed. A point with fixed pressure is

- each input of a producer substation in grid-feeding mode and
- each input and output of a producer substation in grid-forming mode.

In fact, it quickly becomes clear that the pressure holding pump can reach the pressure of an equilibrium state without error only at the entrance of the flow pump. A direct connection via pipes also ensures also fixed pressures at other places in the network [1]. However, as soon as a pump is located within the pipe network along the flow direction, the pressure behind it is changed by this pump. It then depends on the second pump whether a fixed pressure is set without error. In the case of volume flow controllers, an error can appear as Remark 34 shows.

For the controllers, this means that for the producer operated in grid-feeding, the pressure is completely defined only on the side of the pressure holding pump. The capacitor at the output can - due to disturbances or modeling inaccuracies - take any pressure and the flow controller will still work. So if the pressure is to be set to a fixed value, the output must be connected by a pipe coupling to a point of fixed pressure. In the case of customer substation, this goes even further: Here, the pressure may not be equal to a known or desired equilibrium on both sides. Accordingly, it must be connected to a point of fixed pressure on both sides.

This means that the controllers cannot be operated arbitrarily in grid-feeding and grid-forming modes. Instead, pressures must be fixed at strategic points in the grid. At first glance, this may sound like a severe limitation. However, it can also be noted that DHN usually follow a relatively organized structure, as the pipelines follow the street layout in a city [Nus20, p. 72]. Loops are possible, but do not appear initially and do not change the overall treatment. The result is that only in very few places the pressure actually has to be fixed. This is shown for two example networks in Appendix C.3.

> **Remark 32:** The method of using grid-forming controllers strategically becomes more flexible the more producer substations are present in the network. Remember that the term "producer substation" includes all substations that are equipped with a pump for pressure holding. The name only stems from the assumed application area. Hence this is theoretically achievable

[1] With a deviation that can be explained by the pressure loss due to pipe friction

with any network with an arbitrary number of producers, assuming one is willing to bear the installation costs at the consumer substations.

Remark 33: How the modes of operation are determined in practice may vary. Perhaps a continuously available source such as a CHP plant or a geothermal source could always be operated in grid-forming mode. Alternatively, of course, it could be determined rule-based during operation which source is operated in which mode. Since the heat control is usually centralized and requires communication between sources anyway (see Section 1.3.2), this is also easy to realize.

Remark 34: Any controller that controls the volume flow in a pipe has the same behaviour in a steady state: At first, it compensates the pressure differences between the two ends. Then, it adds the exact pressure difference that is needed to produce the desired volume flow.

For a volume flow controller that is located between two storages ($\hat{=}$ elasticities, capacitors, see Figures 3.7 and 3.8), this has a serious implication. It means means that the controller adapts if the storages are differently filled and consequently have a different pressure. It can therefore never determine the pressure at one of the storages, not even in relation to the other one.

This statement can also be reasoned with the observation that one controller cannot control two independent variables.

Remark 35: In practise, the actual pressure at the customer substation should be mainly dependent from the initial conditions. This corresponds to the fact that gas vessels are used which are functioning exactly because of this. Additionally, the capacity of the heat exchanger is moderately high so the influence should not be huge. Considering that only a pressures range and not an exact pressure is needed, the conditions from the beginning of this section could have fewer relevance in practice. This is however a topic of further evaluation.

Remark 36: One could argue that the consideration that makes the conditions necessary has little practical relevance as two capacitors exist in the model, but in reality there is only one heat exchanger. But the same problem appears if only one capacitor is modeled. The difference is just that the constellation capacitor-pump-capacitor appears between different models, not within one.

4.1.3 Design Methodology

Using the results from this chapter and the analysis from the previous one, it can be determined how the controllers should be desinged.

In Sections 3.2.3 and 3.2.4 has been shown that a unified control of pressure and flow at the producer substation is not possible with the examined control schemes. The in Section 3.2.5 proposed alternative was to subdivide the producer into system 1 and system 2 which allows a modular design as follows: The pressure holding controller can operate system 1 the whole time without interruption. In the meantime, the controller for system 2 can switch between grid-feeding and grid-forming mode without affecting the pressure holding.

All things considered, four controllers are needed: The volume flow is controlled in the consumer substations and in system 2 if it is operated in grid-feeding mode. Two controllers are therefore necessary since the two systems are slightly different. Additionally, a controller for system 2 in

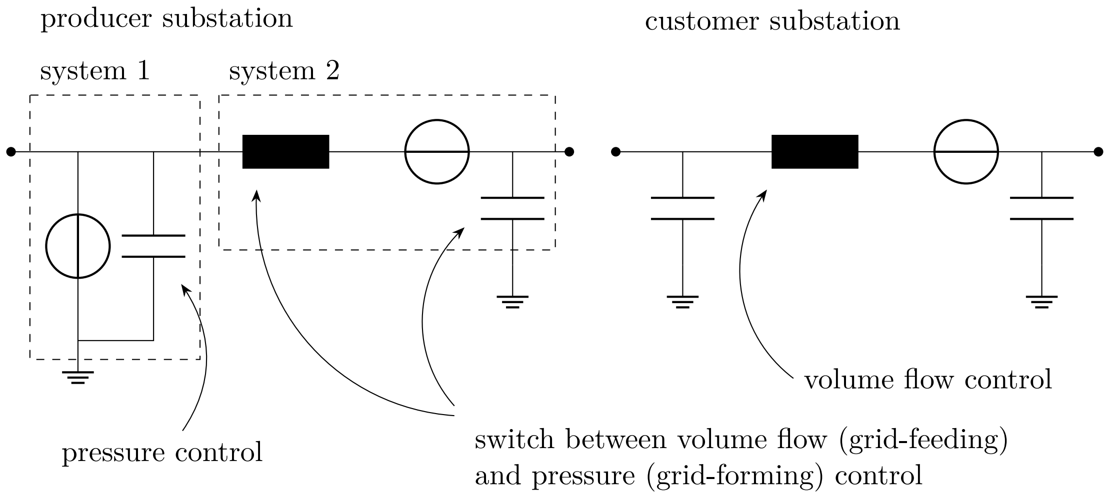


Figure 4.1: Visualisation of which elements and corresponding states are controlled. The ESB is reduced to the relevant elements for the sake of simplicity.

grid-forming mode and a pressure holding controller for system 1 need to be designed. The different controllers resp. control objectives are visualised in Figure 4.1

Knowing the different controllers, it can be proceeded to show how the different controllers are developed.

For the controller designs, the undisturbed, islanded systems are used as defined in Definitions 20 and 21 whose constituting relationships can be given by Eqs. (3.3), (3.20) and (3.21). This means that the $\boldsymbol{d}$-$\boldsymbol{z}$-ports are set to zero during the design step. It allows in accordance with the requirements in Section 1.4 a control law with only local values and influences. This again is necessary to provide the control for networks with variable topologies and independent controllers. Note that the interconnections and disturbances are included later on in the stability analysis in Section 4.3.

System 2 and the consumer substation are controlled with PI-PBC. Due to their natural passivity relation between inputs and outputs, this is possible and it has several practical advantages (ref. Section 2.5.1). First of all, the ease of use suggests PI-controllers. Industrial controller solutions are available which are cheap in acquisition and easy to deploy. Their structure is easy to understand and even engineers with few control experience are able to work with them. Additionally they are very robust and in the following, boundaries are derived within which the parameters can be chosen without endangering the overall stability.

On the downside, the tunig may take some time and they only work with systems that are already passive. The latter is the reason why another method has to be chosen for system 1 and the grid-forming controller. Here, IDA-PBC is selected which is likewise a powerful and popular approach. The particular advantages are the unproblematic handling of the dissipation obstacle (ref. [VJ14, ch. 15.4]), the explicit formulae for the construction and that the results are easy to verify. Since the controlled system becomes a PHS, the desired pressure p_1 becomes a passive output of the system as required in Remark 29.

Because a non-parametrized IDA-PBC usually leads to a matching equation that can be cumbersome to solve, the algebraic IDA-PBC is used. This simplifies the controller design and avoids mathematical hindrances.

4.2 Controller Synthesis

In the following, the controllers for all available pumps are developed. The more extensive design of the IDA-PBC controller of system 1 is conducted at first, followed by the grid-forming controller for system 2, the controller for the consumer substation and for the grid-feeding mode of system 1.

4.2.1 Control of System 1

System 1 has the task to control pressure holding, i.e. the input pressure of the flow pump. The design of the algebraic IDA controller follows the steps presented in Section 2.5.2.

Step 1: Selection of the Hamiltonian At the desired equilibrium for system 1, the pressure over the capacitor C_1 is equal to $p_1 = p_1^*$ and the flow through L_4 is zero, i.e. $q_4^* = 0$. As the states of the system are linearly dependent from these quantities, they follow with Eq. (3.21c) as

$$x_1^* := C_1 p_1^* \tag{4.1a}$$
$$x_4^* := 0. \tag{4.1b}$$

The Hamiltonian of system 1 is a quadratic function and in a slight modification, a desired Hamiltonian can be given as follows:

$$H_d(x_1, x_4) := \frac{(x_1 - x_1^*)^2}{2C_1} + \frac{x_4^2}{2L_4} \tag{4.2}$$

This function is still positive definite as required by Definition 15, but the minimum of the function has shifted to the desired states.

Proof This can be easily checked since $H_d(x_1, x_4)$ is a summation of two shifted quadratic functions. ■

Step 2: Ensure the Definiteness and Symmetry of $\boldsymbol{R}$ and $\boldsymbol{J}$ In order to shape the resulting desired PHS into a valid PHS, the resistive and interconnection structure must fulfill the requirements on the symmetry respectively skew-symmetry and on the definiteness of the matrices. The difference $\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})$ can be expressed using the auxiliary matrix

$$\boldsymbol{F} = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} := \boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x}) \tag{4.3}$$

The original matrices follow as

$$\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{F} = \underbrace{\frac{1}{2}(\boldsymbol{F} - \boldsymbol{F}^\top)}_{\boldsymbol{J}_d(\boldsymbol{x})} - \underbrace{\left(-\frac{1}{2}\right)(\boldsymbol{F} + \boldsymbol{F}^\top)}_{\boldsymbol{R}(\boldsymbol{x})} \tag{4.4}$$

which already ensures the symmetry of $\boldsymbol{R}(\boldsymbol{x})$ and the skew-symmetry of $\boldsymbol{J}_d(\boldsymbol{x})$. In order to satisfy that $\boldsymbol{R}(\boldsymbol{x})$ is positive definite, Sylvester's criterion (Theorem 1) can be used. With

$$\boldsymbol{R}(\boldsymbol{x}) = -\frac{1}{2}(\boldsymbol{F} + \boldsymbol{F}^\top) = \begin{pmatrix} -f_{11} & -f_{12} - f_{21} \\ -f_{21} - f_{12} & -f_{22} \end{pmatrix} \succcurlyeq 0 \tag{4.5}$$

and considering all principal minors of $\boldsymbol{R}(\boldsymbol{x})$, Sylvester's criterion yields

$$f_{11} \leq 0 \tag{4.6a}$$
$$f_{22} \leq 0 \tag{4.6b}$$
$$f_{21} + f_{12} \leq 0 \tag{4.6c}$$
$$\det(\boldsymbol{R}(\boldsymbol{x})) = f_{11}f_{22} - (f_{12} + f_{21})^2 \geq 0 \Leftrightarrow \|f_{12} + f_{21}\| \leq \sqrt{f_{11}f_{22}} \tag{4.6d}$$

The existence of the squareroot in Eq. (4.6d) is always ensured by Eqs. (4.6a) and (4.6b). Eqs. (4.6c) and (4.6d) restrict the sum $f_{12} + f_{21}$ from both sides and the remaining results from this step are

$$f_{11} \leq 0 \tag{4.7a}$$
$$f_{22} \leq 0 \tag{4.7b}$$
$$-\sqrt{f_{11}f_{22}} \leq f_{12} + f_{21} \leq 0. \tag{4.7c}$$

Step 3: Selection of $\boldsymbol{G}^{\perp}$ The selection of $\boldsymbol{G}^{\perp}$ is trivial for system 1. With Eq. (3.20)

$$\boldsymbol{G} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

follows that for any g_1 the relationship

$$\boldsymbol{G}^{\perp}\boldsymbol{G} = \begin{pmatrix} g_1(\boldsymbol{x}) & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4.8}$$

is satisfied. Since $\boldsymbol{G}^{\perp}$ is multiplied on both sides in the ME (2.41), changing $g_1(\boldsymbol{x})$ does not change the result of the ME. Without loss of generality the selection $g_1(\boldsymbol{x}) = 1$ is thus possible.

Step 4: Solving the ME Substituting the previous results and the system parameters into the ME (2.41) gives

$$\begin{aligned} q_4 &= f_{11}\frac{x_1 - x_1^*}{C_1} + f_{12}\frac{x_4}{L_4} \\ &= f_{11}(p_1 - p_1^*) + f_{12}q_4. \end{aligned} \tag{4.9}$$

For a proof, refer to Appendix B.1.

The equation is obviously fulfilled with choosing

$$f_{11} = 0 \tag{4.10a}$$
$$f_{12} = 1 \tag{4.10b}$$

and in combination with Eq. (4.7c) follows

$$0 \leq f_{12} + f_{21} \leq 0 \Leftrightarrow f_{21} = -f_{12} = -1. \tag{4.10c}$$

Hence the desired closed-loop system is according to Eq. (2.44)

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 1 \\ -1 & f_{22} \end{pmatrix} \begin{pmatrix} \frac{x_1 - x_1^*}{C_1} \\ \frac{x_4}{L_4} \end{pmatrix}. \tag{4.11}$$

> **Remark 37:** Note that the interconnection structure is not considered in IDA-PBC but only the islanded model. The actual model including the interconnection structure is
>
> $$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 1 \\ -1 & f_{22} \end{pmatrix} \begin{pmatrix} \frac{x_1 - x_1^*}{C_1} \\ \frac{x_4}{L_4} \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} q_3 \\ q_{\text{in}} \end{pmatrix}. \tag{4.12}$$
>
> This can be verified by inserting the control law into the system equation.

Step 5: Calculation of the IDA-PBC Control Law With these results, the control law of IDA-PBC can be calculated, that is

$$\begin{aligned} \boldsymbol{\beta}(\boldsymbol{x}) &= \boldsymbol{G}^{+}(\boldsymbol{x}) \Big[[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})] \nabla H_d(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}) \Big] \\ &= p_1^* + R_4 \left(\frac{x_4}{L_4} \right) - f_{22} \frac{x_4}{L_4} \end{aligned} \tag{4.13}$$

For a proof refer to Appendix B.1.2.

This controller can drive system 1 already as desired. For additional robustness, an IA is added.

Step 6: Extended targed PHS In system 1, the RD1 state is

$$x_i = x_4 \tag{4.14a}$$

and the HRD state is

$$x_h = x_1. \tag{4.14b}$$

Therefore, the extended target PHS is

$$\begin{pmatrix} \dot{s}_h \\ \dot{s}_i \\ \dot{s}_e \end{pmatrix} = \dot{\boldsymbol{s}} = \begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \partial H_{ds} / \partial s_h \\ \partial H_{ds} / \partial s_i \\ \partial H_{ds} / \partial s_e \end{pmatrix} \tag{4.15}$$

and the new Hamiltonian is

$$\begin{aligned} H_{ds}(\boldsymbol{s}) &= H_d(s_i, s_h) + \frac{K^{-1}}{2} s_e^2 \\ &= \frac{(s_h - x_1^*)^2}{2C_1} + \frac{s_i^2}{2L_4} + \frac{K^{-1}}{2} s_e^2. \end{aligned} \tag{4.16}$$

Step 7: Calculation of the State Transformation The state transformation is found by solving Eq. (2.50) for s_i which yields

$$s_i = \Psi(s_e, x_i, x_h) = x_4 + L_4 K_I K^{-1} s_e. \tag{4.17}$$

The proof can be found in Appendix B.1.3.

With the choice $K_I = K$ this simplifies even more to

$$s_i = \Psi(s_e, x_i, x_h) = x_4 + L_4 s_e = x_i + L_4 s_e. \tag{4.18}$$

The derivative of the transformation is invertible as demanded

$$(\partial^\top \Psi / \partial x_i)^{-1} = (1)^{-1} = 1 \tag{4.19}$$

and therefore satisfies the necessary condition for the existence of the control.

Step 8: Calculation of the IA Control Law The last step of the control procedure is to calculate the IA control law. This means that Eq. (2.51) hast to be solved for $\boldsymbol{v}$. The result is

$$\boldsymbol{v} = L_4 \left(\frac{f_{22}}{L_4} s_e - \dot{s}_e \right) \tag{4.20}$$

which is proven in Appendix B.1.4. Using the lowest row of Eq. (2.48), that is

$$\begin{aligned} \dot{s}_e &= K_I^\top \frac{\partial H_{ds}}{\partial s_h} = K_I \frac{s_h - x_1^*}{C_1} \\ \Leftrightarrow s_e &= K_I^\top \int \frac{\partial H_{ds}}{\partial s_h} \, d\tilde{t} = \int_0^t K_I \frac{s_h - x_1^*}{C_1} \, d\tilde{t} \end{aligned}$$

and Eq. (4.14), the final IA control law can be given as

$$\boldsymbol{v} = \frac{L_4}{C_1} K_I \left(\frac{f_{22}}{L_4} \int_0^t (x_1 - x_1^*) \, d\tilde{t} - (x_1 - x_1^*) \right). \tag{4.21}$$

The complete control law for system 1 can finally be given as follows:

Proposition 9 (IDA-PBC Controller for Pressure Control): The system 1 in a DHN given by Eq. (3.20) has the objective to set the pressure at the input of the flow pump to a certain level. This ensures a safe operation of the flow pump. The system can be controlled with the control law

$$\begin{aligned} u(x) &= \boldsymbol{\beta}(\boldsymbol{x}) + \boldsymbol{v}(\boldsymbol{x}) \\ &= \underbrace{p_1^* + R_4(q_4) - f_{22} q_4}_{\text{IDA-PBC}} + \underbrace{\frac{L_4}{C_1} K_I \left(\frac{f_{22}}{L_4} \int_0^t (x_1 - x_1^*) \, d\tilde{t} - (x_1 - x_1^*) \right)}_{\text{additional IA}}. \end{aligned} \tag{4.22}$$

The two degrees of freedom are $f_{22} \leq 0$ and $K_I \geq 0$.

Proof The proof follows from the previous considerations. ■

The control law can be seen in the following way: IDA-PBC provides an *open-loop* control that sets the pressure directly to the desired value p_1^*. Additionally, the influence of the nonlinear damping $R_4(q_4)$ is compensated which can easily be checked by inserting the control law in the system equations of system 1. The nonlinear damping is replaced by a damping with a freely selectable parameter f_{22}. If this parameter is chosen to be constant, a linear damping results.

The additional IA on the other hand provides a PI-like structure. The first term is the integral part and the second one the proportional one. This interpretation holds only as long as f_{22} is not chosen to be variable, e.g. a function of $\boldsymbol{x}$.

4.2.2 Grid-forming control of System 2

In Remark 31 has been observed that both system 1 and system 2 have the structure of a nonlinear RLC circuit. In the grid-forming mode, the pressure at the output of the producer substation is controlled which means that the pressure over C_2 is the control objective.

Considering the just designed controller for system 1 on a physical level, this controller has done exactly the same thing. It has also controlled the pressure over a capacitor in a nonlinear RLC circuit, in this case the pressure over C_1. Therefore the exact same controller can be used for system 1 as well as system 2 in grid-forming mode.

Proposition 10 (Grid-forming Control of System 2): The control of the grid-forming mode of system 2 can be done in the same way as the control of the pressure maintenance in system 1. The control structure is therefore identical to Proposition 9 and the controller is

$$u(x) = p_2^* + R_3(q_3) - f_{22}q_3 + \frac{L_3}{C_2}K_I\left(\frac{f_{22}}{L_3}\int_0^t (x_2 - x_2^*)\, d\tilde{t} - (x_2 - x_2^*)\right). \qquad (4.23)$$

Proof Compared are Eqs. (3.20) and (3.21) which pose the system equations of system 1 and system 2. IDA-PBC controls the systems in islanded mode, so the ***d*-*z***-ports have to be set to zero. It turns out that both systems have the exact same structure and only the parameters are different. Therefore an IDA-PBC control of system 2 has the exact same result as the control design of the previous section. ∎

Remark 38: For system 1, the design with the direct setting of the pressure seems reasonable from a physical perspective. In the circuit diagram Figure 3.7, one can see that for p_1 to be constant, the flow through the inductance L_4 and the resistor has to be zero in in islanded mode. In this case, system 1 simplifies to a capacitor that is directly connected on both ends with a voltage source. That is why the offset p_1^* is present in the control law. The rest of the control only has to set the errors to zero.

For system 2, the design is not directly intuitive. To see why, it is helpful to consider again the case when only the constant offsets are present in the control laws: Due to the pressure that the pressure control provides, a constant offset in the flow pump would adjust the pressure to a too high level. Specifically it would set it to $p_1^* + p_2^*$ instead of the desired p_2^*.

From intuition, the constant offset in system 2 should thus only be $p_2^* - p_1^*$. In this case, the same idea that explained this control for system 1 would be possible again: The offset sets the desired value and the rest of the control structure eliminates errors. It is clear that this slight unintuitiveness stems intrinsically from IDA-PBC: As the interaction ports stay disregarded, it is impossible to include external influences such as the pressure holding.

However, this is not a problem in practice: The integral action can correct errors and if a constant offset exists, it will diminish over time.

Remark 39: It may be argued that a common control of both pumps would be a better solution. This is in fact possible, but unfavourable. IDA-PBC can control both pressures with a MIMO controller, but as observed in Section 3.2.4, this is not feasible for the flow q_3 and the pressure p_1. Therefore a modular approach is necessary in any case for the grid-feeding mode of the producer.

As stated in Section 4.1, it is therefore still sensible to extend the approach by designing a

SISO grid-forming controller for system 2. This has the further advantage that it enables the switching between grid-feeding and grid-forming mode without affecting the pressure control for the flow pump at the producer.

4.2.3 Control of the Customer Substation

After the design of the two pressure controllers, the controllers for the flow rates are shown. As already announced in Section 4.1, the passivity relationship between the input pressure and the the flow rate can be used. Therefore, a simple PI controller satisfies the control objectives. The control can be given as follows:

Proposition 11: The volume flow in the customer substation as given in Eq. (3.12) can be controlled asymptotically stable with the PI controller

$$\begin{aligned} \dot{\xi} &= -(q_3 - q_3^*) \\ u = p_v &= K_\mathrm{I}\xi - K_\mathrm{P}(q_3 - q_3^*) \end{aligned} \tag{4.24}$$

Proof As the customer substation possesses a PHS structure (see Eq. (3.12)), the relationship between u and y, i.e. between p_v and q_3, is naturally passive. The customer substation can be displayed as a circuit containing only nonlinear RLC elements. The control law is therefore possible and using Proposition 4 closes the proof assuming q_3^* is an admissible equilibrium according to Eq. (2.36). ∎

Remark 40: In islanded mode, the only admissible q_3^* is $q_3^* = 0$. This can easily be checked using Eq. (2.36) and it is logical: The physical limitation that hinders IDA-PBC to set a desired flow rate in islanded mode (ref. Remark 27) stays of course present.

What makes a PI control feasible in contrast to IDA-PBC is something else: In interconnection, a flow rate into and out of the system is possible. This extends the set of admissible equilibria to $q_3^* \neq 0$. Consequently, a flow rate different from 0 does not necessarily imply any more that the pressure over the capacitor changes and constant states are possible.

IDA-PBC in contrast cannot reflect this easily as the PHS structure is designed in islanded mode. Since the control law bases on the PHS structure, no valid control law can be found.

Remark 41: The last remark shows a minor limitation of the control which is obvious from a physical point of view: The set of admissible equilibria is only extended if the flow rate into the system equals the flow rate out of the system. This is of course a behaviour that is desired anyway. It can be achieved by imposing compatible volume flows throughout the DHN.

Remark 42: This control design is the same that has been proposed in [Mac+22] only this year. The difference between the paper and this work is that the pressure levels are also considered in this work here.

4.2.4 Grid-feeding Control of System 2

It has already been observed that the customer substation and system 2 are very similar. In Section 4.1, it was suggested that both systems are controlled by a PI controller. This is in fact possible and the same controller can be used as for the customer substations.

Proposition 12 (Grid-feeding Control of System 2): The grid-feeding control of system 2 as given by Eq. (3.21) can be a PI controller as follows:

$$\begin{aligned} \dot{\xi} &= -(q_3 - q_3^*) \\ u = p_v &= K_\mathrm{I}\xi - K_\mathrm{P}(q_3 - q_3^*) \end{aligned} \tag{4.25}$$

The control law shapes the desired equilibrium q_3^* asymptotically stable.

Proof The proof follows equivalently to the proof of Proposition 11. ∎

Remark 43: The usage of the same controller is plausible since both systems are essentially the same. The only difference is that system 2 lacks the capacitor C_1. The latter is indeed not relevant in this context: When deploying a PI controller, the passivity relationship between input and output is sufficient to show the stability of an equilibrium. As only passivity is needed, it is irrelevant (for the stability of the equilibrium of the controlled variable) how many other states exist in the controlled system.

The results from this section can be regarded as follows: At the beginning of the section, the design of four different controllers for four slightly different systems was demanded. Due to the similarities between the models, it could be shown that in fact only two different controller designs are necessary. They can control all four models.

The pressure can be controlled with an IDA-PBC controller. This controller is deployed for the pressure holding in system 1 and for the grid-forming mode of system 2. Algebraic IDA was used and the design was the most extensive in this section. The volume flow can be controlled easily with a PI controller. This works for the grid-feeding controller of system 2 as well as for the customer substation.

Up to now, the control laws are presented but no proof has been shown that assures the stability of an equilibrium of the whole DHN. All the proofs up to now thus have a preliminary character. In the following section, this missing result is given and the feasibility of the controllers in a DHN is proven.

4.3 Modular Stability Analysis

The modular stability analysis is one of the main contributions of this works. The difficulty lies in the changing network structure as customer substations can plug into and plug out of the network according to the current demand. The producers can likewise plug in and out caused by the volatility of regenerative sources.

To face this challenge, the passivity-based control approach has been chosen that has been mentioned several times throughout this work. It can be used to prove stability.

As the analysis is relatively long, an overview of the single steps is provided beforehand. A visual representation is additionally given in Figure 4.2. In general, the analysis follows the universal thoughts already given in Section 2.3.2.

The first fundamental idea in Section 4.3.1 concerns the passivity of the subsystems: The controller design has been given in the previous Section 4.2. For the pressure controllers, a design with IDA-PBC and additional IA has been designed. The resulting system is a PHS and passivity is assured. Additionally, incremental passivity has to be shown. The volume flows can be controlled with an ordinary PI controller and again incremental passivity needs to be proven.

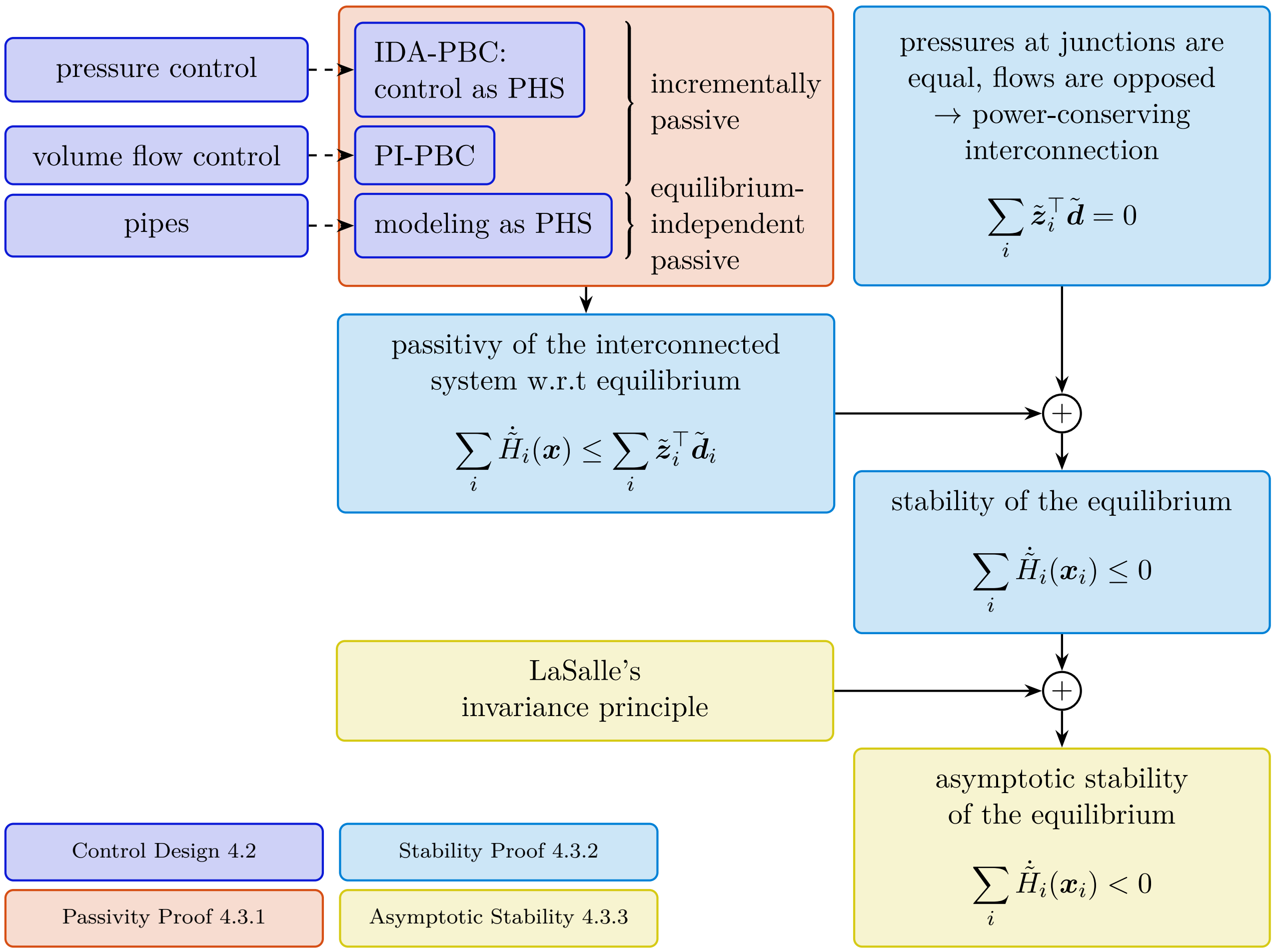


Figure 4.2: The single steps of the modular stability analysis.

The pipes are modeled as (naturally passive) PHSs. They cannot influence their equilibrium actively but the equilibrium is imposed by the neighbouring systems. As the neighbouring systems can choose their equilibrium freely, the equilibrium of the pipe needs to adapt. Normal passivity or incremental passivity is therefore not sufficient. Using them would mean that later on only stability of one distinct equilibrium could be proven. Therefore, EIP is the correct criterion in this case.

After proving the adequate passivity criteria of the subsystems, in Section 4.3.2 the passivity of the interconnected network is derived. In order to derive stability from this observation, the other fundamental idea needs to be used: The power flows between two subsystems are exactly opposed and therefore they cancel each other out when all power flows are summed up. This is the reason why the interconnection structure is power-conserving and the DHN can be seen as a Dirac structure. The passive systems in a Dirac structure form a passive system in interconnection and the interconnected system has no inputs and outputs. Therefore, from passivity of the system follows directly the stability of the equilibrium of the system.

While this argumentation proves stability, it does not prove asymptotic stability. The latter can be shown in the last step in Section 4.3.3 under the use of LaSalle's invariance principle.

4.3.1 Step 1: Passivity of the Subsystems

In a modular stability analysis, the first step is to analyse the features of the single modules. This has to be made for every single subsystem and the calculations therefore become rather long. For this reason, the results are given here but the proofs are postponed to the Appendix C.1.

The first controller that is analysed is the IDA-PBC controller for system 1 and the grid-forming mode of system 2. The closed-loop system of both systems is incrementally passive as given in the following two propositions:

Proposition 13 (Incremental Passivity of System 1): The closed-loop DHN subsystem that is created by controlling system 1 with the control law proposed in Proposition 9 is incrementally passive. The minimum of its energy function is the desired equilibrium $x_1^* = p_1^*/C_1$ and $x_4^* = q_4^*/L_4 = 0$.

Proposition 14 (Incremental Passivity of System 2 in Grid-forming Mode): The closed-loop DHN subsystem that is created by controlling system 2 with the grid-forming control law proposed in Proposition 10 is incrementally passive. The minimum of its energy function is once again the desired equilibrium $x_2^* = p_2^*/C_2$ and $x_3^* = q_3^*/L_3 = 0$.

Proof For a proof of both propositions, see Appendices C.1.1 and C.1.2. ■

Following the proof of [Jay+07], the incremental passivity of the PI controller can be shown. This one is deployed in the consumer substation and in system 2 when operated in grid-feeding mode.

Proposition 15 (Incremental Passivity of the Customer Substation): The closed loop consisting of the customer substation and the PI controller from Proposition 11 is incrementally passive. The minimum of the energy function is at the state with the desired flow rate q_3.

Proposition 16 (Incremental Passivity of System 2 in Grid-feeding Mode): System 2 can be controlled with the grid-feeding controller proposed in Proposition 12. The resulting subsystem is incrementally passive and the energy function is minimal at the desired equilirbium state with the flow rate q_3.

Proof The proofs are given in Appendices C.1.3 and C.1.4. ■

The mentioned propositions analyse the passivity properties of the actively controlled subsystems. In order to proceed with the stability analysis, the last subsystem needs to be considered which is the pipe. In this case, EIP is needed instead of incremental passivity.

Proposition 17 (Equilibrium Independent Passivity of the Pipes): The pipes of a DHN as modeled by Eq. (3.3) are strictly EIP.

Proof This proposition is proven in Appendix C.1.5. ■

This observation finishes the first step of the modular stability analysis. It has been proven that all subsystems of the DHN are passive with respect to their interconnection structure. The energy functions of the actively controlled subsystems are minimal at their particular desired equilibrium. In the case of the pipes, an energy function can be found so that it is - depending from the constant input - minimal at any equilibrium.

4.3.2 Step 2: Passivity and Stability of the Interconnected System

This knowledge can be used to derive the passivity of the network in interconnection as whole.

Proposition 18 (Passivity of the Network in Interconnection): A DHN consisting of pipes, producer and consumer substations is considered. If the subsystems are controlled with the controllers proposed in Section 4.2, it is passive.

Proof The Proof follows fairly straightforward: From Propositions 13 to 17 follows that all controlled subsystems of a DHN are passive w.r.t. their interconnection ports.

According to Section 3.1.4, the DHN is a Dirac structure and therefore the interconnection is power-conserving. Analogous to Proposition 1 a common storage function of the complete system can the be found from the single storage functions $H_i(\boldsymbol{x}_i)$:

$$\tilde{H}_{\text{tot}}(\tilde{\boldsymbol{x}}_{\text{tot}}) = \sum_i \tilde{H}_i(\tilde{\boldsymbol{x}}_i) \geq 0 \tag{4.26}$$

The time derivative of this storage function results in

$$\dot{\tilde{H}}_{\text{tot}}(\tilde{\boldsymbol{x}}) = \sum_i \dot{\tilde{H}}_i(\tilde{\boldsymbol{x}}_i) \leq \sum_i \tilde{\boldsymbol{z}}_i^\top \tilde{\boldsymbol{d}}_i \tag{4.27}$$

and therefore the DHN is passive with respect to $\tilde{\boldsymbol{d}}_i$-$\tilde{\boldsymbol{z}}_i$-ports. ■

Remark 44: It is possible to make this clear from a physical point of view: At any point of interconnection of two or more DHN subsystems, the difference between the pressure at the system ports and the ambient pressure (the "ground") is equal. Additionally, Kirchhoff's current law applies at any connection and the sum of all flows is therefore zero. This means that the power flows between two subsystems are exactly opposed and no energy is lost. Therefore, the system can be considered a Dirac structure.

A physical interpretation of passivity is that energy is never created, but only transported of dissipated. As all subsystems are passive and the interconnection structure does not induce additional energy, the DHN as a whole is equally passive.

Assuming that an equilibrium exists in the interconnection, Proposition 18 also guarantees the stability of this equilibrium:

The DHN is only interconnected within itself and there is no interaction with other systems. Therefore, there are no in- and outputs into the system and from passivity follows Corollary 3.

Corollary (Stability of the Interconnected DHN) A DHN with subsystems at the vertices is considered. Assume that this system is controlled with the controllers proposed in Section 4.2 and an equilibrium exists. Then, the controllers can set the equilibrium and the equilibrium is stable. ■

Proof This follows from the definition of passivity and the fact that the DHN does not interact with other systems. From this follows

$$\sum_i \tilde{\boldsymbol{z}}_i^\top \tilde{\boldsymbol{d}}_i = 0 \tag{4.28}$$

and therefore

$$\dot{\tilde{H}}_{\text{tot}} = \sum_i \dot{\tilde{H}}_i(\boldsymbol{x}) \leq 0 \tag{4.29}$$

the stability of the system. ■

This statement is helpful but not sufficient. On order to control the DHN as desired, asymptotic stability is required. This is achieved in the next section.

4.3.3 Step 3: Asymptotic Stability of the Interconnected System

In order to prove asymptotic stability, LaSalle's invariance principle can be used. The first requirement is stability which has been shown in Corollary 3. The remaining task is to find the invariant set $\mathcal{L}$ with

$$\mathcal{L} \subseteq \{\boldsymbol{x} \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \tag{4.30}$$

Remark 45: In the whole work up to now, by the context it was fairly easy to know which variable belongs to which system. To avoid cluttering the notation, it was therefore refrained from adding indices with information about the corresponding system. As the following calculations need to regard the whole system, a more concise assignment is needed. Consequently, the index c denotes the customer substation. The indices p ("pressure control"), $form$ and $feed$ correspond to system 1 and system 2 in grid-forming respectively grid-feeding mode. The pipes are indexed by l ("line"). $\tilde{q}_{\text{in}}$ and $\tilde{q}_{\text{out}}$ are the volume flows at the interconnection points. As the output of one system is the input of another one, they do not need further indexing.

As the analysis is fairly long, it is in most parts moved to Appendix C.2. The reader may refer to this in case of further interest. In order to show the asymptotic stability of the network, only the results are mentioned at this point.

By setting the derivative of the Hamiltonian to zero and using the system equations, for the DHN subsystems follows

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = \tilde{x}_{\mathrm{p},1} = \tilde{x}_{\mathrm{form},2} = \tilde{x}_{\mathrm{form},3} = \tilde{s}_{\mathrm{form},e} = x_{\mathrm{feed},3} = \tilde{x}_{\mathrm{c},3} = \tilde{x}_{\mathrm{p},4} = 0\}. \tag{4.31}$$

Moreover, some of the states are not necessarily zero but constant. These states are

$$\tilde{p}_{\mathrm{l,\ in}} = \tilde{p}_{\mathrm{l,\ out}} \tag{4.32a}$$
$$\tilde{s}_{\mathrm{p},e} = \tilde{q}_{\mathrm{p},3} = const. \tag{4.32b}$$
$$\tilde{p}_{\mathrm{feed},2} = \tilde{p}_{\mathrm{feed},v} = \tilde{\xi}_{\mathrm{feed}} = const. \tag{4.32c}$$
$$\tilde{p}_{\mathrm{c},1} = const. \tag{4.32d}$$
$$\tilde{p}_{\mathrm{c},2} = const. \tag{4.32e}$$

These results can be obtained by considering only the subsystems themselves. However, this is not yet the complete desired result as also the derivation from pressures should converge to zero. The results up to now can be summarized as follows:

- **Volume flows:** The desired volume flows are always set.
- **Customer:** At the customer substation, the pressures are not necessarily exactly at the desired level as the subsystem cannot control these by itself.
- **Pipes:** The error of the pressure at the input equals the error of the pressure at the output.
- **Producer in Grid-forming Mode:** In grid-forming mode, the pressure at both the input and the output are at the desired value.
- **Producer in Grid-feeding Mode:** The input of every producer substation in grid-feeding mode is exactly at the desired value. At the output, the pressure is constant but the error not necessarily zero.

This observation is equivalent to the prediction that has been made in Section 4.1.2. In the same section, it has already been discussed where the grid-feeding and grid-forming controllers have to be placed throughout the network so that the whole system ist asymptotically stable. For the sake of completeness, these conditions are once again included in the following proposition:

Proposition 19 (Asymptotic Stability of the DHN): Consider a DHN consisting of producer substations, consumer substations and pipes as modeled in Section 3.1. Assume the system is controlled by the controllers presented in Propositions 9 to 12 and assume an equilibrium exists that is to be set. If

- every output of a producer substation in grid-feeding mode and
- all ports of a consumer substation

are connected directly via pipes to

- an input of a producer substation in grid-feeding mode or
- a port of a producer substation in grid-forming mode,

then the asymptotic stability of the desired equilibrium is assured.

Proof It the conditions are satisfied, Proposition 19 can be proven: From Eq. (4.31) and Eq. (4.32a) follows directly that the invariant set $\mathcal{L}$ is

$$\mathcal{L} \subseteq \{\boldsymbol{x} \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} = \{\boldsymbol{x} \in \mathcal{X} | \boldsymbol{x} = \boldsymbol{0}\}. \tag{4.33}$$

Using the stability of the system as proposed in Corollary 3 and LaSalle's invariance principle, i.e. Theorem 3, the asymptotic stability of the equilibrium of the interconnected DHN is proven. ∎

This concludes the modular stability analysis. The designed controllers can set the flows in the whole network to desired values. The pressures can also be adjusted, although certain conditions must be met. Two things shall be underlined at this place:

Firstly, the conditions on the interconnection do not appear not due to poor controller design, but due to the system itself (see Section 4.1.2).

Secondly, the conditions are also not really restrictive, because DHN are designed in a fashion that easily allows to meet them. This analysis exceeds the scope of this chapter, but it can be found in the Appendix C.3.

4.4 Discussion

In this chapter, a possible control structure for the DHN has been proposed. Moreover, it has been shown that the control structure can stabilize a desired equilibrium asymptotically. The difficulty for both the controller design as well as the stability analysis was the changing network structure in accordance with Section 1.4. Therefore, not only the model had to be considered during the design step, but also its interaction with possibly connected systems. No direct proof of stability could be made but the modular stability analysis was necessary.

The most important characteristic of the control - i.e. the stabilizing property - has already been discussed exhaustively in the previous section. Nevertheless, some additional observations on the developed control law can be made.

The first one is that both the traditional control law as explained in Section 1.3.1 and the newly developed one need a producer that fixes the pressure level. Both of them have a pressure control pump and a determined pressure difference between input and output. Therefore, they set the absolute pressure levels at the ports as well as the pressure difference between the ports.

The difference is that in traditional control, the pressure difference is the actual goal while the setting of the absolute pressures is a byproduct. In the newly proposed control, it is exactly opposite. Additionally of course, the here proposed control law ensures passivity.

One advantage that both control laws share is that the actual desired flows at the customer substations do not have to be known. Instead, they can be set at the customer substation individually as noted in the following remark.

Remark 46: The calculation for the stability analysis ensures that the flow exactly meets the equilibrium in grid-forming mode. However, the equilibrium flow of the producer itself is not further specified and *unknown*. This is an advantage, as the customers can demand any input flow that is necessary to heat the building and the producer will follow at any time. The explanation is as follows:

Interpret the junction at the output of the producer in grid-forming mode as the junction that closes the whole network. This junction then has one input flow that comes from the producer

> and multiple output flows that go to the customers. At this junction, the pressure is fixed by the input of the junction and the flows are fixed by the outputs. The compatibility constraint from Section 2.4.2 is therefore automatically satisfied for the whole network. Moreover, it is satisfied independently from the desired values of the single controllers.
>
> In other words and without extensive proof, under normal conditions the existence of the desired equilibrium should be guaranteed at any time.

The next observation concerns the IDA-PBC controller that has been developed for the pressure control. If the terms that linearize the dissipation term in the equation are ignored, the remaining control law (4.22) can be interpreted as a PI controller in combination with a feedforward control. In the last years, some efforts have been made to prove the applicability of PI controllers in nonlinear and passive control [Jay+07; BON18; BOS21]. The results focused on the control of input-output-combinations with a passive behaviour. But in the case of the controller in this work, the control law is functioning even if the controlled system is not passive with respect to the used input and output. This observation could be generalizable. Maybe even more easy-to-understand PI-like control laws exist if only a constant offset is added to the control law.

Another point where a generalization of the results from this work seems feasible is for the control of other energy networks. The controller developed in [Str+20] for an electric DC microgrid has the same structure as the IDA-PBC controller in this thesis. This is because both models of the energy network are RLC-circuits. Since both use IDA-PBC and IA, the resulting controllers are identical. As the models in different domains of energy distribution are similar, a unified approach for the control of energy networks could be possible.

This concludes the main part of this thesis. In this chapter, a controller design for 4GDHN and 5GDHN has been proposed and the asymptotic stability of the network has been shown. Additionally, some remarks on the controller design has been made. Therefore, the Challenges C1 to C4 have been solved and the objectives O1 to O4 derived from them have been met.

Chapter 5

Conclusion and Outlook

In this thesis, future developments of DHN and the transition towards 4GDHN and 5GDHN are analysed. The main focus are on the implications that this evolution has for the control of DHN.

According to the analysis, challenges arise both in the thermal and the hydraulic domain that must be resolved to achieve the full potential of DHNs. While active research and development is happening in the field of thermal control for DHNs, the field of hydraulic control is still underdeveloped. Especially the handling of the vastly increased number of active pumps in the network is an open task. Additionally, the continuously changing network structure with a plug in and plug out of both heat producers and heat consumers needs to be treated.

Although a small number of theoretical works exist that are dedicated to this task, they have the fundamental disadvantage. They do not consider the pressures in the network which play an important role for a safe operation. If the pressures are not in a certain range, the network can be damaged and additional maintenance is necessary. Therefore, gap in research exists that this thesis wants to close. In a first step, existing models of DHN are extended. The models in this work can represent the pressures in the network and the corresponding pressure control pumps. The modeling uses a port-Hamiltonian approach that can be perpetuated later on in the control design. There, the closed-loop systems can equally be seen as PHSs.

In a second step, appropriate control laws are found that can assure on the one hand the desired volume flows and therefore the desired heat distribution. On the other hand, they guarantee under some conditions on the network structure also a safe operation and suitable pressure levels. To take the volatility of regenerative sources and the changing demand of the customers into account, a modular approach is used.

The control design is based on a control strategy taken from the control of electric microgrids. The producers are controlled in one of two modes, either the grid-feeding or the grid-forming mode. In grid-feeding mode, the volume flows are controlled and therefore directly the amount of heat that is provided to the network. In grid-forming mode in contrast, safe pressure levels are ensured. This is to the best of the authors knowledge the first time that such an approach has been used for the hydraulic control of DHN.

The control is based on passivity features and allows for a plug in and plug out of single substations. The asymptotic stability of a desired equilibrium can be proven with this approach. Therefore, the results from this work can be regarded as one of the first practical proposals for the control of next generation DHNs.

In future works, an extensive simulation of the developed controllers is necessary. This is not included in this thesis in order to reduce the workload to a reasonable level. If all simulations work as expected, an application in a real world DHN would be desirable. The step from the theoretical treatment to the actual application, however, needs additional research and cannot be done easily.

Another task for further reseach could be to reexamine the models. Possibly, the compressibility of the water could be included in the models. This could be reasonable as air can appear in the pipes which increases the elasticity. Under circumstances, the elasticity of the heat exchanger could also be modeled as one capacitor instead of two. This may be a better representation of the underlying physics.

On a more general level, it would be interesting under which conditions the IDA-PBC control law becomes similar to a PI control. This stands in close relation to the question under which conditions a PI control law can be deployed on non-passive input-output pairs.

To summarize, this thesis contributes to the development of a sustainable energy production. The presented controllers can regulate the hydraulic level of DHNs and thus guarantee a desired and safe operation.

Appendix A

Backgrounds of the Modeling

For the main part of the thesis which is the design of a suitable control, an extended modeling section is not necessary. For interested readers or a later revision of the models is could be albeit interesting. Therefore, this section of the appendix provides some additional information over the modeling.

A.1 Fluid Dynamics - a Short Overview

In order to describe the relevant relationships, this section provides a brief summary over fluid dynamics. In this field, the most general description of physical effects are the Navier-Stokes equations [Zie22, p. 160]. Unfortunately, they are very difficult to solve in general cases and therefore scientists and engineers have found some effective simplifications to handle the complexity.

A.1.1 Flow filament theory

One of these simplifications is the flow filament theory [Zie22, p.57 ff.]. It states that under some conditions the flow does not have to be regarded in three dimensions, but a treatment as a one-dimensional variable is possible. In this case, the fluid flows along a so-called flow filament and across the profile of this filament the flow variables velocity, pressure, density and temperature stay the same. In pipes, the necessary assumptions are mostly given and this work treats the problems in terms of flow filament theory.

A.1.2 Conservation laws

Along a filament, two important conservation laws hold: The first one is the conservation of mass which states that the mass flow along a filament is constant [Zie22, p. 93]:

$$\dot{m} = \rho A q = const.$$

with the mass m, the density ρ, the velocity q, and the area of the filament profile A. This holds especially for the connections of multiple pipes where the mass flow in the node and the mass flow out of the node are the same and in sum zero. In the case of constant density and pipe diameters, it follows

$$\sum q_{in} = \sum q_{out}. \tag{A.1}$$

This is also known as the generalized version of Kirchhoff's current law [VJ14, p. 126].

The second conservation law is the Euler equation for fluid dynamics which follows from the momentum conservation. It comprises all forces acting within a flow filament and can be written as [Zie22, p. 94]

$$\frac{\partial q}{\partial t} + q\frac{\partial q}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dy}{ds}$$

introducing the gravity on earth g, the height y and the static pressure p.

In the case of pipe flows, some assumptions can simplify the expression. The pipe diameter is normally constant and together with the law of conservation of mass and A1, it follows that the fluid velocity at the entrance is as fast as the fluid velocity at the output. Consequently follows $\frac{\partial q}{\partial s} = 0$.

Additionally the influence of the height becomes zero if the whole pipe lies on the same elevation. This is equivalent to assumption A5 and can still be made if the actual network has height differences. One can justify this with the observation that in the latter case the height differences stay the same over the whole time of operation. After all, the pipes are mounted in the ground and cannot be moved. Accordingly, only constant offsets emerge that clutter the notation without providing additional information.

Under these two assumptions, the Euler equation can be written in a simplified form as

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} \tag{A.2}$$

A.1.3 Reynolds number

In order to describe different flow characteristics, hydraulic engineers have developed a theory called similarity theory. It uses dimensionless quantities which give information about how the flow will behave.

In the case of pipe friction, the important behaviour is whether the flow is laminar or turbulent. Laminar flow means that the macroscopically observable flow is ordered in several parallel layers. This is the case in flow filament theory. In contrast, turbulent flow is unsteady and unordered. Vortex-like random motion can occur and for an observer, the future fluid movements are not easily predictable without mathematical calculations [Zie22, p. 140].

The dimensionless quantity characterizing the flow type is called the Reynolds number. It is defined as

$$\mathrm{Re} = \frac{qd}{\nu}$$

with the diameter of the pipe d and the kinematic viscosity ν [Zie22, p. 138 ff.].

If the Reynolds number is smaller than 2300, the flow can be seen as laminar. If it is bigger, there exists a transition zone with transient flow until the flow becomes fully turbulent. In the case of the transient zone, a quasi-laminar calculation is possible for pipes if the calculations are executed for the averages [Zie22, p. 145].

A.1.4 Pipe friction

The pressure loss in pipe systems is a quadratic function of the velocity. It can be given as

$$\Delta p = \frac{\rho}{2} q^2 \zeta \tag{A.3}$$

The parameter ζ represents a loss coefficient which can be expressed as

$$\zeta = \frac{l}{d} f$$

if the pipe is hydraulically smooth (ref. [Zie22, p. 147]). Here, l represents the length of the pipe and f is the friction factor. The latter, again, depends on the Reynolds number $f = g(\text{Re})$. Note that f is sometimes also called λ in literature, but λ is used slightly differently in this work.

The connection between Re and f is not continuous and multiple formulae exist depending on the range of Re. In the case of pipe friction, the law of Colebrook and White gives the desired relation [Lin17, p. 7]. It is given in a formulation matching to the previous notation by

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon}{d \cdot 3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

The formula is therefore implicit. The solutions are always positive and consequently it is possible to lump all parameters into one positive parameter

$$\lambda := \frac{\rho}{2} \frac{l}{d} f \geq 0.$$

Considering that the pressure loss points always in the inverse direction of the flow, it is necessary to modify Eq. (A.3) slightly with the sign function[1]. It then can be reformulated as

$$\Delta p = \text{sgn}(q) \lambda q^2. \tag{A.4}$$

This relationship is also depicted in Figure A.1. It is clear from the visual conception and can also be proven that the function is monotone.

This concludes the summary of fluid dynamics. The key elements relevant for the rest of the work are, as presented in Section 3.1.5 and the following, Kirchhoff's generalized current law, the Euler equation and the expression for pipe friction.

[1] Otherwise of course, $\Delta p \sim q^2$ would mean that for negative q, flow and pressure would direct in the same direction - instead of representing an energy loss, the friction would insert power into the system. The widely used formulation Eq. (A.3) is thus not meant to be used in the context of changing flow directions.

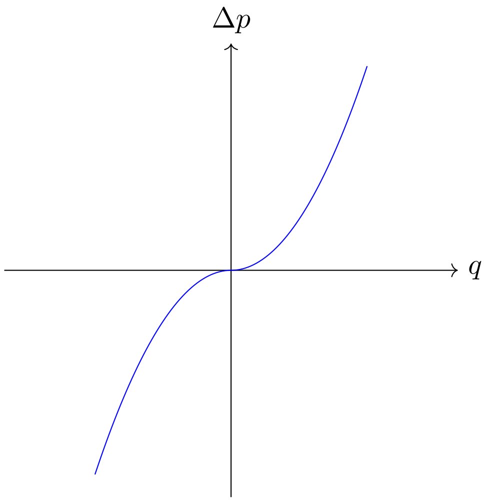


Figure A.1: The relationship between the flow q in a pipe and the pressure difference Δp due to resistance at the openings.

Appendix B

Proofs for the Controller Design

The following sections show the missing proofs that have been omitted during the controller design.

B.1 Design of System 1

B.1.1 Solution of the ME

In Section 4.2.1, the proof of the ME (2.41) has been omitted. Using the ME, the system equations of system 1 (Eq. (3.20)) and the results from the previous steps yields

$$
\begin{aligned}
\boldsymbol{G}^{\perp}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) &= \boldsymbol{G}^{\perp}(\boldsymbol{x})[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})]\nabla H_d(\boldsymbol{x}) \\
\Leftrightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \nabla H(\boldsymbol{x}) - \begin{pmatrix} 0 \\ R_4(q_4) \end{pmatrix} \right] &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} \begin{pmatrix} \frac{x_1 - x_1^*}{C_1} \\ \frac{x_4}{L_4} \end{pmatrix} \\
\Leftrightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} q_4 \\ -p_1 - R_4(q_4) \end{pmatrix} &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} f_{11} \frac{x_1 - x_1^*}{C_1} + f_{12} \frac{x_4}{L_4} \\ f_{21} \frac{x_1 - x_1^*}{C_1} + f_{22} \frac{x_4}{L_4} \end{pmatrix} \\
\Leftrightarrow q_4 &= f_{11} \frac{x_1 - x_1^*}{C_1} + f_{12} \frac{x_4}{L_4} \\
&= f_{11}(p_1 - p_1^*) + f_{12} q_4
\end{aligned}
\tag{B.1}
$$

which finishes the proof.

B.1.2 IDA-PBC Control Law

The calculation of the control law can be given as follows, using Eq. (3.20) and the resulting system (4.11):

$$
\begin{aligned}
\boldsymbol{\beta}(\boldsymbol{x}) &= \boldsymbol{G}^{+}(\boldsymbol{x}) \Big[[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})]\nabla H_d(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}) \Big] \\
&= \left[\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0 \end{pmatrix} \left[\begin{pmatrix} \frac{x_4}{L_4} \\ -\frac{x_1 - x_1^*}{C_1} + f_{12} \frac{x_4}{L_4} \end{pmatrix} - \begin{pmatrix} q_4 \\ -p_1 - R_4(q_4) \end{pmatrix} \right] \\
&= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{x_1^*}{C_1} + f_{12} \frac{x_4}{L_4} + R_4(q_4) \end{pmatrix} \\
&= \frac{x_1^*}{C_1} + f_{12} \frac{x_4}{L_4} + R_4(q_4) \\
&= p_1^* + R_4 \left(\frac{x_4}{L_4} \right) - f_{22} \frac{x_4}{L_4}
\end{aligned}
\tag{B.2}
$$

B.1.3 IA State Transformation

Using the extended target PHS (4.15) from the previous step and the system to be controlled (4.11), the state transformation can be calculated. This is equal to solve Eq. (2.50) for s_i:

$$\begin{aligned}(\boldsymbol{J}_{hi} - \boldsymbol{R}_{hi})\frac{\partial H_d}{\partial \boldsymbol{x}_i} + (\boldsymbol{J}_h - \boldsymbol{R}_h)\frac{\partial H_d}{\partial \boldsymbol{x}_h} &= (\boldsymbol{J}_{hi} - \boldsymbol{R}_{hi})\frac{\partial H_{ds}}{\partial s_i} + (\boldsymbol{J}_h - \boldsymbol{R}_h)\frac{\partial H_{ds}}{\partial s_h} - \boldsymbol{K}_I\frac{\partial H_{ds}}{\partial s_e} \\ 1\frac{x_4}{L_4} + 0\frac{x_i - x_1^*}{C_1} &= 1\frac{s_i}{L_4} + 0\frac{s_h - x_1^*}{C_1} - K_I K^{-1} s_e \\ \frac{x_4}{L_4} &= \frac{s_i}{L_4} - K_I K^{-1} s_e \\ \Leftrightarrow \qquad s_i &= \Psi(s_e, x_i, x_h) = x_4 + L_4 K_I K^{-1} s_e\end{aligned} \tag{B.3}$$

B.1.4 IA Control Law

The control law can be found by solving Eq. (2.51) for the control input $\boldsymbol{v}$:

$$\begin{aligned}&(\boldsymbol{J}_i - \boldsymbol{R}_i)\frac{\partial H_{ds}}{\partial s_i} + (\boldsymbol{J}_{ih} - \boldsymbol{R}_{ih})\frac{\partial H_{ds}}{\partial s_h} = \\ &\qquad \frac{\partial^\top \Psi}{\partial \boldsymbol{x}_i}\left[(\boldsymbol{J}_i - \boldsymbol{R}_i)\frac{\partial H_d}{\partial \boldsymbol{x}_i} + (\boldsymbol{J}_{ih} - \boldsymbol{R}_{ih})\frac{\partial H_d}{\partial \boldsymbol{x}_h} + \boldsymbol{G}_i(\boldsymbol{x})\boldsymbol{v}\right] + \frac{\partial^\top \Psi}{\partial \boldsymbol{x}_h}\dot{\boldsymbol{x}}_h + \frac{\partial^\top \Psi}{\partial s_e}\dot{s}_e \\ \Leftrightarrow \qquad & f_{22}\frac{s_i}{L_4} - 1\frac{s_h - x_1^*}{C_1} = 1\left[f_{22}\frac{x_4}{L_4} - 1\frac{x_1 - x_1^*}{C_1} + \boldsymbol{v}\right] + 0\dot{x}_h + L_4 K_I K^{-1}\dot{s}_e \\ \Leftrightarrow \qquad & \boldsymbol{v} = L_4 K_I K^{-1}\left(\frac{f_{22}}{L_4}s_e - \dot{s}_e\right)\end{aligned}$$

Appendix C

Analysis of Modular Stability

C.1 Incremental Passivity

Due to the lengthy calculations, the proof for the passivity of the single systems has been postponed from Section 4.3.1 to this section of the appendix.

C.1.1 Incremental Passivity of System 1

Following the order of Section 4.3.1, the first system treated is system 1. In Proposition 13, it has been said that the closed loop subsystem created by controlling system 1 with the control law proposed in Proposition 9 is incrementally passive. Additionally, the minimum of its energy function is the desired equilibrium $x_1^* = p_1^*/C_1$ and $x_4^* = q_4^*/L_4 = 0$.

In order to prove this, consider the following: Take the extended target PHS of the IA step Eq. (4.15), its Hamiltonian Eq. (4.16) and consider the interaction ports.

$$\begin{pmatrix} \dot{s}_h \\ \dot{s}_i \\ \dot{s}_e \end{pmatrix} = \begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{s_h - x_1^*}{C_1} \\ \frac{s_i}{L_4} \\ K^{-1} s_e \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} q_3 \\ q_{\text{in}} \end{pmatrix} \tag{C.1}$$

Define the incremental variables

$$\tilde{\boldsymbol{s}} = \boldsymbol{s} - \boldsymbol{s}^* \tag{C.2}$$

and let the Hamiltonian of the incremental system follow as

$$\begin{aligned} \tilde{H}_{d\boldsymbol{s}}(\tilde{\boldsymbol{s}}) &= \frac{(s_h - 0)^2}{2C_1} + \frac{(s_i - \bar{s})^2}{2L_4} + \frac{K^{-1}}{2}(s_e - \bar{s}_e)^2 \\ &= \frac{\tilde{s}_h^2}{2C_1} + \frac{\tilde{s}_i^2}{2L_4} + \frac{K^{-1}}{2}\tilde{s}_e^2 \end{aligned} \tag{C.3}$$

with using the known equilibrium $s_h^* = 0$. Note that the constant term x_1^* disappears in $\tilde{s}_h$ as it is present both in s_h and s_h^*[1].

The incremental system itself can easily be given by deriving Eq. (C.2) and using the state equations Eq. (C.1) as

$$\dot{\tilde{\boldsymbol{s}}} = \begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{s}_h/C_1 \\ \tilde{s}_i/L_4 \\ K^{-1}\tilde{s}_e \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_3 \\ \tilde{q}_{\text{in}} \end{pmatrix} \tag{C.4}$$

[1] This is the reason why the known and desired equilibrium is $s_h^* = 0$ instead of $s_h^* = x_1^*$.

Thus the incremental passivity relation can be shown

$$\begin{aligned}
\dot{\tilde{H}}_{d\boldsymbol{s}}(\tilde{\boldsymbol{s}}) &= \nabla \tilde{H}_{d\boldsymbol{s}}^\top(\tilde{\boldsymbol{s}})\dot{\tilde{\boldsymbol{s}}} \\
&= \begin{pmatrix} \tilde{s}_h/C_1 \\ \tilde{s}_i/L_4 \\ K^{-1}\tilde{s}_e \end{pmatrix}^\top \left[\begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{s}_h/C_1 \\ \tilde{s}_i/L_4 \\ K^{-1}\tilde{s}_e \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_3 \\ \tilde{q}_{\text{in}} \end{pmatrix} \right] \\
&= f_{22} \left(\frac{\tilde{s}_i}{L_4} \right)^2 + \frac{\tilde{s}_h}{C_1}(\tilde{q}_3 - \tilde{q}_{\text{in}}) \\
&= f_{22} \left(\frac{\tilde{s}_i}{L_4} \right)^2 + \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \\
&\leq \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \qquad \text{with } f_{22} \leq 0
\end{aligned} \tag{C.5}$$

which concludes the proof.

C.1.2 Incremental Passivity of System 2 in Grid-forming Mode

The same consideration can be made for system 2: According to Proposition 14, the closed-loop subsystem created by controlling system 2 (defined by Eq. (3.21)) with the grid-forming control law (proposed in Proposition 10) is incrementally passive. The minimum of its energy function is the desired equilibrium $x_2^* = p_2^*/C_2$ and $x_3^* = q_3^*/L_3 = 0$.

The incremental passivity can be shown exactly as in the previous section. For the sake of completeness, it is added here anyway.

Take the extended target PHS of the IA step and consider the interaction ports.

$$\begin{pmatrix} \dot{s}_h \\ \dot{s}_i \\ \dot{s}_e \end{pmatrix} = \begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{s_h - x_2^*}{C_2} \\ \frac{s_i}{L_3} \\ K^{-1}s_e \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} q_{\text{out}} \\ p_1 \end{pmatrix} \tag{C.6}$$

with the Hamiltonian

$$\begin{aligned}
H_{d\boldsymbol{s}}(\boldsymbol{s}) &= H_d(s_i, s_h) + \frac{K^{-1}}{2} s_e^2 \\
&= \frac{(s_h - x_2^*)^2}{2C_2} + \frac{s_i^2}{2L_3} + \frac{K^{-1}}{2} s_e^2.
\end{aligned} \tag{C.7}$$

Define the incremental variables

$$\tilde{\boldsymbol{s}} = \boldsymbol{s} - \boldsymbol{s}^* \tag{C.8}$$

and let the Hamiltonian of the incremental system follow as

$$\begin{aligned}
\tilde{H}_{d\boldsymbol{s}}(\tilde{\boldsymbol{s}}) &= \frac{(s_h - 0)^2}{2C_2} + \frac{(s_i - \bar{s})^2}{2L_3} + \frac{K^{-1}}{2}(s_e - \bar{s}_e)^2 \\
&= \frac{\tilde{s}_h^2}{2C_2} + \frac{\tilde{s}_i^2}{2L_3} + \frac{K^{-1}}{2}\tilde{s}_e^2
\end{aligned} \tag{C.9}$$

with using the known equilibrium $s_h^* = 0$. Note that the constant term x_2^* disappears in $\tilde{s}_h$ as it is present both in s_h and s_h^*.[2]

[2] This is the reason why the known and desired equilibrium is $s_h^* = 0$ instead of $s_h^* = x_2^*$.

The incremental system itself can easily be given by deriving Eq. (C.8) and using the state equations Eq. (C.6) as

$$\dot{\tilde{\boldsymbol{s}}} = \begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{s}_h/C_2 \\ \tilde{s}_i/L_3 \\ K^{-1}\tilde{s}_e \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_{\text{out}} \\ \tilde{p}_1 \end{pmatrix} \tag{C.10}$$

Thus the incremental passivity relation can be shown

$$\begin{aligned} \dot{\tilde{H}}_{\boldsymbol{ds}}(\tilde{\boldsymbol{s}}) &= \nabla \tilde{H}_{\boldsymbol{ds}}^\top(\tilde{\boldsymbol{s}})\dot{\tilde{\boldsymbol{s}}} \\ &= \begin{pmatrix} \tilde{s}_h/C_2 \\ \tilde{s}_i/L_3 \\ K^{-1}\tilde{s}_e \end{pmatrix}^\top \left[\begin{pmatrix} 0 & 1 & -K_I \\ -1 & f_{22} & 0 \\ K_I & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{s}_h/C_2 \\ \tilde{s}_i/L_3 \\ K^{-1}\tilde{s}_e \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_{\text{out}} \\ \tilde{p}_1 \end{pmatrix} \right] \\ &= f_{22} \left(\frac{\tilde{s}_i}{L_3} \right)^2 - \frac{\tilde{s}_h}{C_2}\tilde{q}_{\text{out}} + \frac{\tilde{s}_i}{L_3}\tilde{p}_1 \\ &= f_{22} \left(\frac{\tilde{s}_i}{L_3} \right)^2 + \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \\ &\leq \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \qquad\qquad \text{with } f_{22} \leq 0 \end{aligned} \tag{C.11}$$

which concludes the proof.

C.1.3 Incremental Passivity of the Customer Substation

Following the proof of [Jay+07], the incremental passivity of the consumer substations can be shown. As stated in Proposition 15, the closed loop consisting of the customer substation Eq. (3.12) and the PI controller from Proposition 11 is incrementally passive. The minimum of the energy function is at the state with the desired flow rate q_3.

In [Jay+07], only the stability of the desired state is shown. The same approach, however, can also prove that the resulting closed-loop system is incrementally passive with regard to the interconnection structure.

The incremental customer substation can be calculated with a state vector of known and unknown equilibria $\bar{\boldsymbol{x}}^*$

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - \bar{\boldsymbol{x}}^* \tag{C.12}$$

as

$$\dot{\tilde{\boldsymbol{x}}} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \tilde{q}_3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ R_3(q_3) - R_3(q_3^*) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tilde{p}_v + \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_{\text{in}} \\ \tilde{q}_{\text{out}} \end{pmatrix} \tag{C.13}$$

and the Hamiltonian of the incremental closed-loop system is

$$\begin{aligned} \tilde{H}_{\text{cl}}(\tilde{\boldsymbol{x}}) &= H(\tilde{\boldsymbol{x}}) + \frac{K_{\text{I}}}{2}\xi^2 \\ &= \frac{\tilde{x}_1^2}{2C_1} + \frac{\tilde{x}_2^2}{2C_2} + \frac{\tilde{x}_3^2}{2L_3} + \frac{K_{\text{I}}}{2}\xi^2 \\ &= \frac{(x_1 - x_1^*)^2}{2C_1} + \frac{(x_2 - \bar{x}_2)^2}{2C_2} + \frac{(x_3 - \bar{x}_3)^2}{2L_3} + \frac{K_{\text{I}}}{2}\xi^2. \end{aligned} \tag{C.14}$$

The derivative of the Hamiltonian $\tilde{H}_{\text{cl}}(\tilde{\boldsymbol{x}})$ is

$$\begin{aligned}
\dot{\tilde{H}}(\tilde{\boldsymbol{x}}) &= \nabla\tilde{H}(\tilde{\boldsymbol{x}})\dot{\tilde{\boldsymbol{x}}} + K_{\text{I}}\xi\dot{\xi} \\
&= \begin{pmatrix}\tilde{p}_1\\ \tilde{p}_2\\ \tilde{q}_3\end{pmatrix}^{\top} \left[\begin{pmatrix}0 & 0 & -1\\ 0 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}\tilde{p}_1\\ \tilde{p}_2\\ \tilde{q}_3\end{pmatrix} - \begin{pmatrix}0\\ 0\\ R_3(q_3)-R_3(q_3^*)\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\tilde{p}_v + \begin{pmatrix}1 & 0\\ 0 & -1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{q}_{\text{in}}\\ \tilde{q}_{\text{out}}\end{pmatrix}\right] + K_{\text{I}}\xi\dot{\xi} \\
&= -\tilde{q}_3(R_3(q_3)-R_3(q_3^*)) + \tilde{q}_3\tilde{p}_v + \tilde{p}_1\tilde{q}_{\text{in}} - \tilde{p}_2\tilde{q}_{\text{out}} + K_{\text{I}}\xi\dot{\xi}
\end{aligned} \tag{C.15}$$

Using the definition of the PI controller

$$\begin{aligned}
\dot{\xi} &= -(q_3 - q_3^*) \\
u = p_v = K_{\text{I}}\xi - K_{\text{P}}(q_3 - q_3^*) \Leftrightarrow \tilde{\xi} \quad &= \frac{1}{K_{\text{I}}}(\tilde{p}_v + K_{\text{P}}\tilde{q}_3)
\end{aligned} \tag{C.16}$$

gives in combination with Eq. (C.15)

$$\begin{aligned}
\dot{\tilde{H}}(\tilde{\boldsymbol{x}}) &= -\tilde{q}_3(R_3(q_3)-R_3(q_3^*)) + \tilde{q}_3\tilde{p}_v + \tilde{p}_1\tilde{q}_{\text{in}} - \tilde{p}_2\tilde{q}_{\text{out}} - \tilde{q}_3K_{\text{I}}\frac{1}{K_{\text{I}}}(u + K_{\text{P}}\tilde{q}_3) \\
&= -\tilde{q}_3(R_3(q_3)-R_3(q_3^*)) - \frac{K_{\text{P}}}{K_{\text{I}}}\tilde{q}_3^2 + \tilde{p}_1\tilde{q}_{\text{in}} - \tilde{p}_2\tilde{q}_{\text{out}} \\
&= -\tilde{q}_3(R_3(q_3)-R_3(q_3^*)) - \frac{K_{\text{P}}}{K_{\text{I}}}\tilde{q}_3^2 + \tilde{\boldsymbol{d}}^{\top}\tilde{\boldsymbol{z}} \\
&\leq \tilde{\boldsymbol{d}}^{\top}\tilde{\boldsymbol{z}}
\end{aligned} \tag{C.17}$$

The last inequality holds because of the monotonicity of the resistive structure (see Eq. (3.2)) and as $K_{\text{P}}, K_{\text{I}} > 0$ is required by definition.

Therefore, the closed-loop system is incrementally passive and with Eq. (C.14) follows, that the desired equilibrium is at the minimum of the energy function.

C.1.4 Incremental Passivity of System 2 in Grid-feeding Mode

The same approach can be used to prove the passivity of system 2 ind grid-feeding mode. This system operated in a closed-loop together with the grid-feeding controller proposed in Proposition 12 is once again incrementally passive. The energy function is minimal at the desired equilirbium state.

The proof follows directly from the proof of Proposition 15: System 2 is the same as the customer substation only with the missing capacitor C_1. Therefore, this proposition can be proven by using the previous proof and setting $x_1 = C_1p_1 = 0$.

C.1.5 Equilibrium-independent Passivity of the Pipes

The mentioned propositions analyse the passivity properties of the actively controlled subsystems. In order to proceed with the stability proof, the last subsystem needs to be considered, which is the pipe. Following Proposition 17, the pipes of a DHN as modeled by Eq. (3.3) are strictly EIP.

For a system to be EIP, the two mappings $k_x : \mathcal{U}^* \rightarrow \mathcal{X}$ and $k_y : \mathcal{U}^* \rightarrow \mathcal{Y}$ must exist that allocate a unique u^* and y^* to every x_l^*. $k_x(u^*)$ can be found using the state equation (3.3d):

$$0 \stackrel{!}{=} \dot{x}_l = -\underbrace{\lambda_l(q_l)}_{=\lambda_l(\frac{x_l}{L})} + \underbrace{p_{\text{in}} - p_{\text{out}}}_{=:p_{\text{tot}}} \tag{C.18}$$

The two inputs p_{in} and p_{out} can be subsumed under the single input p_{tot}. As $\lambda_l(q_l)$ is a monotonically increasing function and a unique mapping (see Eq. (3.3a)), Eq. (C.18) proves the existing of a unique mapping $k_x(u^*) = k_x(p_{\text{tot}})$ for all $p_{\text{tot}}^* \in \mathbb{R}$.

The first row of Eq. (3.3e) is a linear map from x_l to y and therefore also a unique map and strictly monotonically increasing function $k_y(x_l^*)$ exists for all $x_l^* \in \mathbb{R}$.

Using this, a proceeding similar to the proof of incremental passivity can be chosen: The incremental system with the variables $\tilde{x}_l = x_l - \bar{x}_l$ possesses the Hamiltonian

$$\tilde{H}(\tilde{x}_l) = \frac{\tilde{x}_l^2}{2L} \tag{C.19}$$

and the state equation

$$\dot{\tilde{x}}_l = -R(q_l) - (-R(\bar{q}_l)) + \tilde{p}_{\text{in}} - \tilde{p}_{\text{out}}. \tag{C.20}$$

The time derivative of the Hamiltonian is

$$\begin{aligned} \dot{H}(\tilde{x}_l) &= \nabla H(\tilde{x}_l)\dot{\tilde{x}}_l \\ &= \frac{\tilde{x}_l}{L}(-R(q_l) - (-R(\bar{q}_l)) + \tilde{p}_{\text{in}} - \tilde{p}_{\text{out}}) \\ &= -(q_l - \bar{q}_l)(-R(q_l) + R(\bar{q}_l)) + \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \\ &< \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \qquad\qquad \forall x_l = Lq_l \neq 0. \end{aligned} \tag{C.21}$$

Since the monotonically increasing function k_x exists and due to Eq. (C.21), the system is strictly passive with respect to every constant input $p_{\text{tot}}^* = p_{\text{in}}^* - p_{\text{out}}^*$. This is equal to strict EIP.

C.2 Invariant Sets

The determination of invariant sets for the application of LaSalle's invariance principle needs a rather long calculation. Therefore it has also been postponed to the appendix.

Following Section 4.3.3, in this section indices are used to distinguish between the variables of different systems. They are labelled as described in Remark 45.

The task is to find the largest invariant set $\mathcal{L}$ contained in

$$\{\boldsymbol{x} \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\}, \tag{C.22}$$

i.e. $\mathcal{L} \subseteq \{\boldsymbol{x} \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\}$.

The analysis is structured as follows: First of all, the pipes are considered as all subsystems are connected via the pipes. Afterwards, the single subsystems are regarded and the results from the pipes are included. It turns out that system 1 and system 2 can only be treated together. This is not a problem as they are both contained in the producer substation and they will not appear in the DHN individually.

C.2.1 Pipes

In the case of the pipes, the invariant set looks as follows: Eq. (C.22) restricts the states to the single state

$$\tilde{x}_l = \tilde{q}_l = 0. \tag{C.23}$$

This follows directly by Eq. (C.21). The pipe as modeled by (3.3d) has only one state and therefore follows $\tilde{x}_l = 0 \Rightarrow \dot{\tilde{x}}_l = 0$. This means $\{\tilde{x}_l = 0\}$ is an invariant set and therefore

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = 0\}. \tag{C.24}$$

Regarding Eq. (3.3d) gives the additional condition on the variables at the interconnection point

$$\tilde{q}_{\text{in}} = \tilde{q}_{\text{out}} = 0. \tag{C.25}$$

The absolute pressure levels at the input and the output of the pipes cannot be directly deduced - after all, the volume flow in a pipe is created by a pressure difference and not by absolute pressures. But the system equations allows to say that if the pressure at one end of the pipe is known and the volume flow is known, the pressure at the other end can be determined. If the pressure at one end is constant, it follows

$$\tilde{p}_{\text{l, in}} = \tilde{p}_{\text{l, out}}. \tag{C.26}$$

C.2.2 Producer Substation

As already mentioned, the producer substation can only be analysed by combining insights from system 1 and system 2. First of all, system 1 will be considered and afterwards system 2 in both operating modes. At the end, the last statements about system 1 can be made.

System 1 For the analysis, the controlled system has to be regarded. Using the condition $\dot{H}_\text{p}(\boldsymbol{x}_\text{p}) = 0$ (see Eq. (C.22)) and the derivative of the Hamiltonian Eq. (C.5), this gives directly

$$\tilde{s}_{\text{p},i} = 0. \tag{C.27}$$

The rest of the conditions need to be derived from the system equation Eq. (4.15). By applying the last result and Eq. (C.27), the system equation simplifies to

$$\begin{pmatrix} \dot{s}_{\text{p},h} \\ 0 \\ \dot{s}_{\text{p},e} \end{pmatrix} = \begin{pmatrix} -K_I K^{-1} \tilde{s}_{\text{p},e} + \tilde{q}_{\text{p},3} \\ -\tilde{s}_{\text{p},h}/C_1 \\ \tilde{s}_{\text{p},h}/C_1 \end{pmatrix} \tag{C.28}$$

and therefore also

$$\tilde{s}_{\text{p},h} = \tilde{x}_{\text{p},1} = 0. \tag{C.29}$$

For now, this only means

$$\tilde{s}_{\text{p},e} = \tilde{q}_{\text{p},3} = const., \tag{C.30}$$

a result that will be refined later on.

System 2 in in Grid-forming Mode With $\dot{H}_{\text{form}}(\boldsymbol{x}_{\text{form}}) = 0$ and Eq. (C.11) follows equivalently to system 1

$$\tilde{s}_{\text{form},i} = 0. \tag{C.31}$$

The system equation equation can be used and the conditions at the system ports are $\tilde{x}_{\text{p},1} = 0$ (ref. Eq. (C.29)) and Eq. (C.25). This yields

$$\tilde{s}_{\text{form},h} = \tilde{x}_{\text{form},2} = 0 \tag{C.32}$$
$$\tilde{s}_{\text{form},e} = 0 \tag{C.33}$$

and using the state transformation also

$$\frac{x_{\text{form},3}}{L_3} = q_{\text{form},3} = 0. \tag{C.34}$$

The complete state equation becomes zero and therefore Eq. (C.22) yields

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = \tilde{x}_{\text{p},1} = \tilde{x}_{\text{form},2} = \tilde{x}_{\text{form},3} = \tilde{s}_{\text{form},e} = 0\}. \tag{C.35}$$

System 2 in in Grid-feeding Mode The derivative of the Hamiltonian of system 2 controlled in grid-feeding mode becomes

$$\dot{\tilde{H}}_{\text{feed}}(\tilde{\boldsymbol{x}}_{\text{feed}}) = -\tilde{q}_{\text{feed},3}(R_3(q_{\text{feed},3}) - R_3(q^*_{\text{feed},3})) - \frac{K_{\text{P}}}{K_{\text{I}}}\tilde{q}^2_{\text{feed},3} + \tilde{\boldsymbol{d}}^\top \tilde{\boldsymbol{z}} \tag{C.36}$$

and therefore also for the grid-feeding mode follows

$$\frac{x_{\text{feed},3}}{L_3} = q_{\text{feed},3} = 0. \tag{C.37}$$

This applied on the control law (4.25) gives

$$\tilde{p}_{\text{feed},v} = \tilde{\xi}_{\text{feed}} = const. \tag{C.38}$$

and together with the system equation (3.21) it follows also

$$\tilde{p}_{\text{feed},2} = \tilde{p}_{\text{feed},v} = \tilde{\xi}_{\text{feed}} = const. \tag{C.39}$$

This yields

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = \tilde{x}_{\text{p},1} = \tilde{x}_{\text{form},2} = \tilde{x}_{\text{form},3} = \tilde{s}_{\text{form},e} = x_{\text{feed},3} = 0\}. \tag{C.40}$$

C.2.3 Consumer Substation

For the customer substation,

$$\tilde{q}_{\text{c},3}L_3 = \tilde{x}_{\text{c},3} = 0 \tag{C.41}$$

follows due to the derivative of the Hamiltonian Eq. (C.17). Under consideration of the system equation (3.12a) and the control law (4.24), additionally follows

$$\begin{aligned} \tilde{x}_{\mathrm{c},1} &= const. \\ \tilde{x}_{\mathrm{c},2} &= const. \\ \tilde{p}_{\mathrm{c},v} &= \frac{\tilde{x}_{\mathrm{c},1}}{C_1} - \frac{\tilde{x}_{\mathrm{c},2}}{C_2} = const. \end{aligned} \tag{C.42}$$

and therefore

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = \tilde{x}_{\mathrm{p},1} = \tilde{x}_{\mathrm{form},2} = \tilde{x}_{\mathrm{form},3} = \tilde{s}_{\mathrm{form},e} = x_{\mathrm{feed},3} = \tilde{x}_{\mathrm{c},3} = 0\}. \tag{C.43}$$

C.2.4 Considering the Network Structure

Both the grid-feeding and the grid-forming controller restrict the flow $\tilde{q}_3$ to zero. The state transformation (4.18) of system 1 therefore also yields

$$\tilde{x}_{\mathrm{p},4} = 0 \tag{C.44}$$

and

$$\mathcal{L} \subseteq \{x \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\} \subseteq \{x \in \mathcal{X} | x_l = \tilde{x}_{\mathrm{p},1} = \tilde{x}_{\mathrm{form},2} = \tilde{x}_{\mathrm{form},3} = \tilde{s}_{\mathrm{form},e} = x_{\mathrm{feed},3} = \tilde{x}_{\mathrm{c},3} = \tilde{x}_{\mathrm{p},4} = 0\}. \tag{C.45}$$

This can be used for a tentative result: Using the stability of the network as shown in Corollary 3, LaSalles's invariance principle can be used. This means that the states will converge to the largest invariant set $\mathcal{L} \in \{\boldsymbol{x} \in \mathcal{X} | \dot{H}(\boldsymbol{x}) = 0\}$. Together with Eq. (C.45), this means that the proposed controller structure can set all volume flows in the network at any point of time to the desired equilibrium.

However, this is not yet the complete desired result as also the derivation from pressures should converge to zero. At the moment it can only be shown, that they stay constant. The extended result can be given if the structure of the network is included. There are two variants that can appear:

- **Grid-feeding Mode:** Eq. (C.45) ensures that the pressure $\tilde{p}_{\mathrm{p},1} = \tilde{x}_{\mathrm{p},1}/C_1$ at the input of every producer substation is exactly at the desired value. Therefore, the pressures at both sides of the to the input connected pipes are equally at the equilibrium point (see Eq. (C.26)). In grid-feeding mode, this statement cannot be made at the output of the producer substation.
- **Grid-forming Mode:** In grid-forming mode, the pressure at the input-side of the producer substation is at the desired point with an argumentation as for the grid-feeding mode. Additionally, Eq. (C.32) resp. Eq. (C.45) assure the pressure at the output to be error-free. Again, the connected pipes are equally at the equilibrium. In consequence, all pipes that are connected to a producer substation operated in grid-forming mode are at the desired equilibrium.

This finishes the calculations and the proof for Eq. (4.31) and Eq. (4.32a).

C.3 Practical Relevance of Conditions on the Network Structure

The structure of the DHN changes only with the plug in and plug out of producers and consumers. The pipes in contrary are permanently installed in the ground and conduct water all the time in the

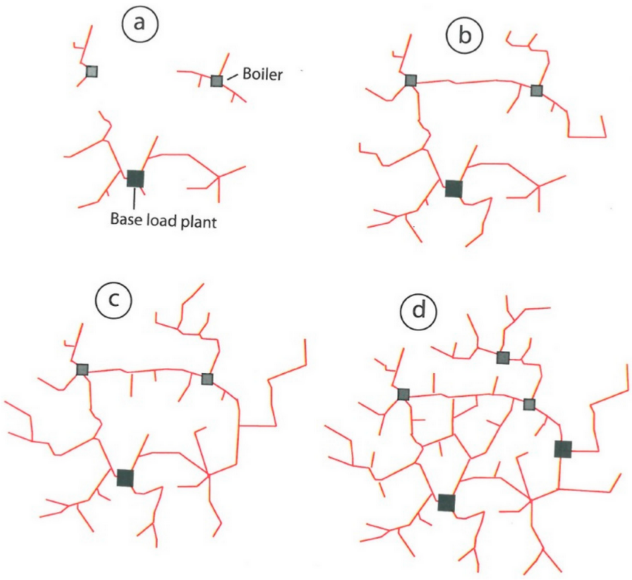


Figure C.1: The four stages of a DHN. Taken from [VT17].

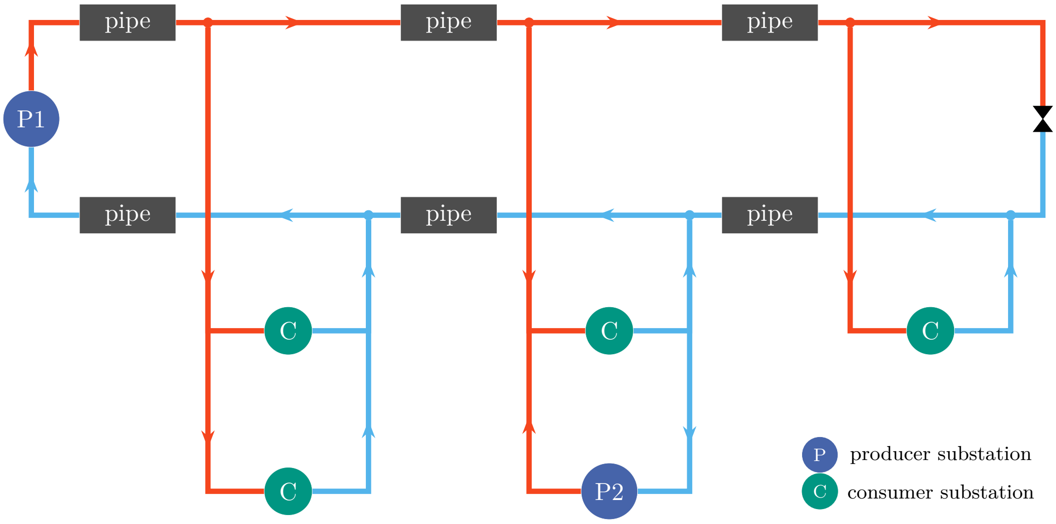


Figure C.2: DHN with tree structure. Operating P1 or P2 in grid-forming mode ensures the asymptotic stability in the whole network.

same direction. Therefore, the network structure does not change significantly during operation. In addition, the pathway of the pipes and the design of the connections of the individual substations to the network are uniform and there are few deviations from the structure. The conditions can therefore be easily met in ordinary DHNs.

According to [VT17], the structure of a DHN can be classified as one of four stages. Growing from small to big, it starts with separated islands and evolves in a tree structure. In bigger networks a ring structure and finally a mesh structure is visible. The four stages are displayed in Figure C.1.

Essentially, the separated islands also have a tree structure and the mesh structure is just a more interconnected ring structure. In the following, two small examples thus are considered. The first one has a tree structure as structure (b) in Figure C.1 and the other one has a mesh structure as in (d). The latter example is taken from [Str+].

In Figure C.2, the network has a tree structure and it has two producers and several consumers.

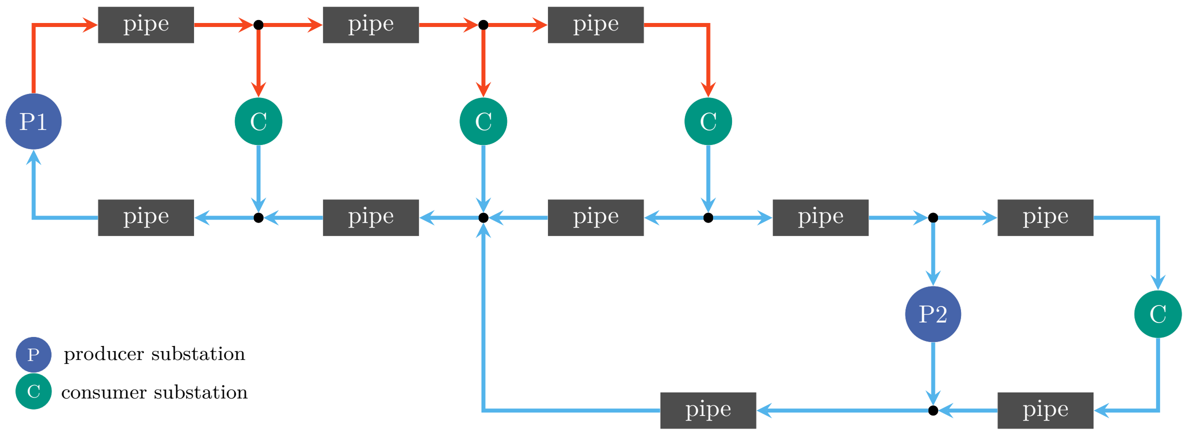


Figure C.3: DHN with mesh structure. Operating P1 in grid-forming mode ensures the asymptotic stability in the whole network. Adapted from [Str+].

There are two uninterrupted layers of pipes which conduct either warm water (red) or cold water (blue). All of the producers and consumers have a connection to both of the layers on each side.

This means that one of the two producers has to be operated in grid-forming mode. Then, every consumer is connected through pipes without interruption directly to the grid-forming producer. Also the other producer is connected at every input and every output to the grid-forming producer and the pressures are fixed, regardless of whether it is operated in grid-forming or grid-feeding mode. According to Proposition 19, asymptotic stability is then ensured.

In other words: Fulfilling the requirements in this example (and by implication every tree structure) boils down to operating one producer in grid-forming mode.

> **Remark 47:** If this example is interpreted physically, the stability analysis is in some way reformulated: If one of the two producers is operated in grid-forming mode, then the pressure at the input and the output of this substation is fixed. Additionally, the controllers always ensure fixed volume flows. If the pressure at one side of a pipe and the volume flow through it is known, the pressure at the other side is also known. This means that the pressure at every point in the two pipe layers is fixed. As the two pipe layers border every substation, the pressure at every substation is also fixed.

The other network is displayed in Figure C.3 and is taken from [Str+]. It is a simple example of a network with a mesh structure. Although the original source regards the network as a network in three layers, one could also regard it as a network with two pipe layers. In fact, two uninterrupted pipelines can be found. Following the first example, these are again colored red and blue[3].

In this network, not every producer is connected to the red layer. Between P2 and the red layer always lies another substation. Therefore, operating P2 in grid-forming mode could not ensure asymptotic stability of an equilibrium.

On the other hand, P1 is connected to both pipe layers and also directly to all inputs and outputs. If in contrary this producer is operated in grid-forming mode, all pressures in the whole network are fixed.

For this example, the conditions on the operation modes are a bit stricter as it is necessary to operate P1 in grid-forming mode. In bigger meshed networks, two things can change: Firstly, with

[3]If one considers that P2 is connected to the blue layer on both sides, it becomes clear that the color here has nothing to do with the water temperature any more. However, the color allows to distinguish between the two layers anyway.

another pipe layer maybe a second producer has to operate in grid-forming mode. And secondly, a growing number of producers in the network should allow more flexibility for choosing the operation modes.

The two examples can represent the different stages of DHNs that can appear. In both stages, operation modes of the producers could be found in a way that satisfies the conditions of Proposition 19. As the network structure does not differ strongly between different DHNs, also for other networks it should be possible to satisfy the conditions. The conditions therefore have a rather mild characteristic and do not really impair the results of this work.

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